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ACYCLIC 3-CHOOSABILITY OF PLANAR GRAPHS WITH NO CYCLES OF LENGTH FROM 4 TO 11

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Abstract. Every planar graph is known to be acyclically 7-choosable and is conjectured to be acyclically 5-choosable (Borodin et al., 2002). This conjecture if proved would imply both Borodin's acyclic 5-color theorem (1979) and Thomassen's 5-choosability theorem (1994). However, as yet it has been verified only for several restricted classes of graphs. Some sufficient conditions are also obtained for a planar graph to be acyclically 4- and 3-choosable.

In particular, a planar graph of girth at least 7 is acyclically 3-colorable (Borodin, Kostochka and Woodall, 1999) and acyclically 3-choosable (Borodin et al., 2010). A natural measure of sparseness, introduced by Erdős and Steinberg, is the absence of k-cycles, where $4 \le k \le C$. Here, we prove that every planar graph with no cycles of length from 4 to 11 is acyclically 3-choosable.

Keywords: acyclic coloring, planar graph, forbidden cycles

1. Introduction

By $V(G)$ and $E(G)$ denote the sets of vertices and edges of a graph G, respectively. The girth of G, i.e. the length of a shortest cycle in G, is denoted by $g(G)$.

A (proper) *k-coloring* of G is a mapping $f : V(G) \longrightarrow \{1, 2, \dots, k\}$ such that $f(x) \neq f(y)$ whenever x and y are adjacent in G.

By the Grötzsch theorem, every planar triangle-free graph is 3-colorable. In 1976, Steinberg conjectured that every planar graph without 4-cycles and 5-cycles

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is 3-colorable. This conjecture remains unsettled. Erdös (see [25]) suggested the following relaxation of this problem: does there exist a constant C such that the absence of cycles with size from 4 to C in a planar graph guarantees its 3-colorability? The best result in this direction is $C \le 7$ due to Borodin et al. [12].

Now suppose each vertex v of a graph G is given a list $L(v)$ of admissible colors, represented by positive integers. The G-list L is *choosable* if there is a proper vertex coloring of G such that a color of each vertex v belongs to $L(v)$. A graph G is said to be k*-choosable* if every G-list L is choosable provided that $|L(v)| \ge k$ for each $v \in V(G)$.

Thomassen [26] proved a famous theorem that each planar graph is 5-choosable, and Voigt [28] showed that this bound is best possible. Margit Voigt constructed the following non-3-choosable planar graphs: of girth at least 4 (in [29]), and without 4-cycles and 5-cycles (in [30]). On the other hand, it is known that a planar graph is 3-choosable if its girth is at least 5 (Thomassen [27]) or it has no cycles of length from 4 to 9 (Borodin [2]).

A proper vertex coloring of a graph is *acyclic* if every cycle uses at least three colors (Grünbaum [19]). Borodin [1] proved Grünbaum's conjecture that every planar graph is acyclically 5-colorable. This bound is best possible; moreover, there are bipartite 2-degenerate planar graphs G which are not acyclically 4-colorable (Kostochka and Mel'nikov [21]). Acyclic colorings turned out to be useful in obtaining results about other types of colorings; for a survey see monographs [20, 17].

Borodin et al. [9] proved that every planar graph is acyclically 7-choosable and conjectured a common extension of Borodin's [1] and Thomassen's results [26]:

Conjecture 1. *Every planar graph is acyclically 5-choosable.*

However, this challenging conjecture seems to be difficult. As yet, it has been verified only for several restricted classes of planar graphs. Some sufficient conditions are also obtained for a planar graph to be acyclically 4- and 3-colorable or choosable. The minimum k with the property that a graph G is acyclically k-colorable (acyclically k-choosable) is denoted by $a(G)$ (by $a^l(G)$).

In particular, Borodin, Kostochka and Woodall [8] showed that if G is a planar graph of girth g then $a(G) \le 4$ if $g \ge 5$ and $a(G) \le 3$ if $g \ge 7$. Note that the first of these results is best possible in terms of girth due to the construction in [21]. Borodin proved $a(G) \le 4$ for G having neither 4- nor 6-cycles (in [3]) and neither 4- nor 5-cycles (in [4]).

Recently, $a^l(G) \le 4$ was proved in the following cases: if $g \ge 5$ (Montassier [22]), or if G has no 4-, 5- and 6-cycles, or no 4-, 5- and 7-cycles, or no 4-, 5- and intersecting 3-cycles (Montassier, Raspaud and Wang [23]), or no 4-, 5- and 8-cycles (Chen and Raspaud [15]), or no 4-, 6- and 7-cycles, or else no 4-, 6- and 8-cycles (Chen, Raspaud and Wang [16]), or neither 4-cycles nor 6-cycles adjacent to a triangle (Borodin, Ivanova, and Raspaud [10]). Note that [10] obviously covers most of the above mentioned results in [8, 22, 23, 16, 3].

Moreover, $a^l(G) \le 3$ was proved if $g \ge 7$ (Borodin et al. [6]) or if G has no cycles of length from 4 to 12 (Borodin [5] and, independently, Hocquard and Montassier [18]).

The purpose of this paper is to improve the result in [5, 18] as follows:

Theorem 2. *Every planar graph with no cycles of length from 4 to 11 is acyclically 3-choosable.*

The choosability version of Steinberg-Erdös problem is to find the smallest C^l such that each planar graph without cycles of length from 4 to C^l is 3-choosable. Borodin [2] proved that $C^l \leq 9$ and Voigt [30] proved that $C^l \geq 6$. It seems natural to ask a similar question about the acyclic 3-choosability:

Problem 1. *Find the smallest C_a^l such that each planar graph without cycles of length from 4 to C_a^l is acyclically 3-choosable.*

It follows from [30] and Theorem 2 that $6 \leq C_a^l \leq 11$. Since the acyclic 3-choosability is a stronger property of a graph than the proper 3-choosability, one can expect that $C_a^l > 6$. Note that Borodin's bound $C^l \leq 9$ proved in [2] remains the best known for over fifteen years, and the proof of Theorem 2 is non-trivial, so we would like to ask the following cautious question:

Problem 2. *Is it true or not that each planar graph without cycles of length from 4 to 10 is acyclically 3-choosable?*

A distinctive feature of our proof of Theorem 2 is that a charge of vertices can be transferred along "feeding paths" to an unlimited distance. This kind of "global" discharging was introduced by Borodin, Ivanova, and Kostochka in [13] and used in [13, 14] to improve results in [7, 11] about homomorphisms of sparse graphs to the circulant $C(5; 1, 2)$ and cycle C_5.

2. Proof of Theorem 2

Suppose a graph G^* with a list L is a counterexample to Theorem 2 on the fewest vertices. Clearly, G^* is connected and has no 1-vertices.

If G^* is 2-connected, then we put $G^+ = G = G^*$. Otherwise, let G^+ be a pendant block, w^+ be the cut-vertex of G^* that belongs to G^+, and f^+ be the face of G^+ that is not a face of G^*. Clearly, w^+ belongs to the boundary of f^+. Now we put $G = G^+ - w^+$.

By $F(G^+)$, $d(v)$, and $r(f)$ denote the set of faces of G^+, the degree of a vertex v in G^+, and the size of face f in G^+, respectively.

From Euler's formula $|V(G^+)| - |E(G^+)| + |F(G^+)| = 2$, using well-known relations

$$\sum_{v \in V(G^+)} d(v) = 2|E(G^+)| = \sum_{f \in F(G^+)} r(f),$$

we have

$$\sum_{v \in V(G^+)} (5d(v) - 12) + \sum_{f \in F(G^+)} (r(f) - 12) = -24.$$

We set the *initial charge* of every vertex $v \in V(G^+) - w^+$ and face $f \in F(G^+)$ to be $ch(v) = 5d(v) - 12$ and $ch(f) = r(f) - 12$, respectively. If w^+ exists, then we put $ch(w^+) = 5d(w^+) + 11$.

Note that only 2-vertices and 3-faces of G^+ except for vertex w^+ have a negative initial charge. Then we use a discharging procedure leading to a *final charge* ch^* such that

$$\sum_{x \in V(G^+) \cup F(G^+)} ch^*(x) = \sum_{x \in V(G^+) \cup F(G^+)} ch(x) < 0. \tag{1}$$

Based on the structural properties of G, we shall get a contradiction with (1) by proving that $ch^*(x) \geq 0$ for every $x \in V(G^+) \cup F(G^+)$.

2.1. **Structural properties of the minimum counterexample.** Recall that all degrees are considered in G^+ and observe that $d(w^+) \geq 2$ since G^+ is a pendant block.

Lemma 1. *No 2-vertex v in G belongs to a 3-cycle.*

Proof. It suffices to acyclically color the graph $G^* - v$ according to list L, and then color v differently from its neighbors. □

Lemma 2. *G has no two adjacent 2-vertices.* □

A *triplet* is a 3-face incident with three 3-vertices of G. (So, a triplet is not incident with w^+ if w^+ exists.) A *garland* (see Fig. 1) is a non-empty sequence of triplets $\mathcal{G}_k = T_1, \ldots, T_k$, where $T_i = x_i y_i z_i$, $1 \leq i \leq k \geq 1$, such that z_i is adjacent to x_{i+1} whenever $1 \leq i \leq k-1$, while x_1 is adjacent to a 2-vertex $x_1' \neq w^+$. The neighbor of x_1' not belonging to T_1 is denoted by x_1''. By y_i' denote the neighbor of y_i such that $y_i' \notin T_i$, where $1 \leq i \leq k$, and let z_k' be the neighbor of z_k lying outside T_k.

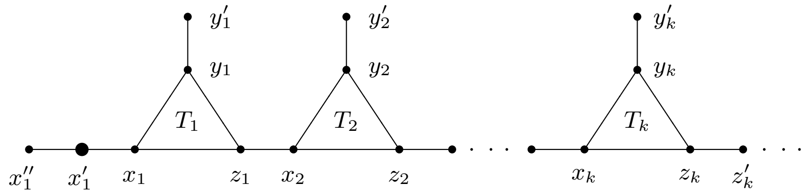

Fig. 1. Garland $\mathcal{G}_k$

The following lemma shows the key idea in the proof of Theorem 2.

Lemma 3. *Suppose $\mathcal{G}_k$ is a garland, c is an acyclic L-coloring of $G^* - x_1'$, and $L(x_1') = \{1,2,3\}$; then*
(i) w.l.o.g., $c(x_1'') = 1$,
(ii) each of the $3k$ vertices of $\mathcal{G}_k$ has the same set $\{1,2,3\}$ of admissible colors, and
(iii) $c(z_k') = c(y_i') = 1$ whenever $1 \leq i \leq k$.

Proof. Induction on k.

STEP 1. We are easily done if $c(x_1) \neq c(x_1'')$, so suppose $c(x_1) = c(x_1'')$. If $c(x_1'') > 3$ then it suffices to put $c(x_1') \in \{1,2,3\} \setminus \{c(y_1), c(z_1)\}$, so we can assume that $c(x_1'') = 1$ and there are bicolored $(1,2)$- or $(1,3)$-cycles in G if we put $c(x_1') = 2$ or $c(x_1') = 3$, respectively. By symmetry, we can assume that $c(y_1) = 2$, $c(z_1) = 3$ and $c(y_1') = c(x_2) = 1$.

If, say, $4 \in L(x_1)$ then it suffices to color x_1 with 4. If $4 \in L(y_1)$ then we can recolor y_1 with 4 and put $c(x_1') = 2$. Similarly, if $4 \in L(z_1)$ then we can recolor z_1 with 4 and put $c(x_1') = 3$. So, $L(x_1) = L(y_1) = L(z_1) = \{1,2,3\}$.

STEP $i+1$, $1 \leq i \leq k-1$. Suppose $L(v) = \{1,2,3\}$ whenever $v \in T_1 \cup T_2 \cup \ldots \cup T_i$, and $c(v) = 1$ whenever $v \in \{y_1', \ldots, y_i', z_i'\}$.

Let a coloring c_2 be obtained from c by putting $c_2(x_1) = \ldots = c_2(x_i) = 1$, $c_2(x'_1) = c_2(z_1) = \ldots = c_2(z_i) = 2$, and $c_2(y_1) = \ldots = c_2(y_i) = 3$; this can create only a $(2,1)$-path through x'_1. A coloring c_3 is defined by swapping numbers 2 and 3 on $\{x'_1\} \cup T_1 \cup T_2 \cup \ldots \cup T_i$ in coloring c_2.

Arguing as in Step 1, we see from c_2 and c_3 that: (a) $c(x_{i+1}) = 1$, $\{c(y_{i+1}), c(z_{i+1})\} = \{2,3\}$, (b) $c(y'_{i+1}) = c(x_{i+2}) = 1$, where $x_{k+1} = z'_k$, and (c) $L(x_{i+1}) = L(y_{i+1}) = L(z_{i+1}) = \{1,2,3\}$. □

Corollary 1. *No garland can close on itself; more specifically, none of vertices y'_k, z'_k in $\mathcal{G}_k$ can coincide with one of $y_1, \ldots, y_{k-1}$.*

Proof. It suffices to note (see Fig. 2) that $c(y'_k) = c(z'_k) = 1$ while none of $y_1, \ldots, y_{k-1}$ is colored 1 due to Lemma 3. □

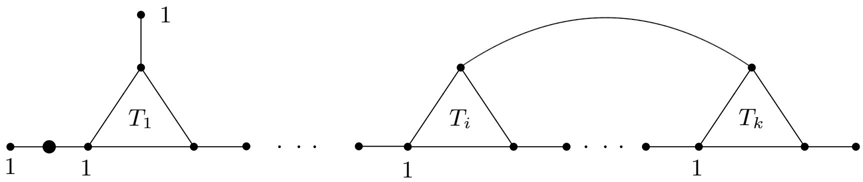

Fig. 2. Closing of FP: $y'_k = y_i$, where $i < k$

Lemma 4. *At most one edge can join a garland to a 2-vertex of G.*

Proof. Let a garland $\mathcal{G}_k$ be the smallest counterexample. This means that none of $y'_1, \ldots, y'_{k-1}$ is a 2-vertex. By symmetry between y'_k and z'_k, suppose that $d(z'_k) = 2$, where vertices x'_1 and z'_k may be distinct or coincide (see Fig. 3). Note that z'_k is distinct from y'_k due to the absence of adjacent 3-cycles in G and from $y'_1, \ldots, y'_{k-1}$ by the assumption just made.

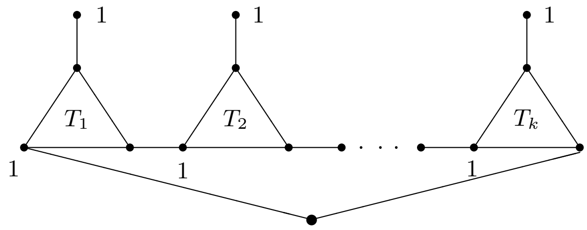

Fig. 3. Closing of FP: $z'_k = x'_1$

Take the coloring c_3 described in the proof of Lemma 3 (in which all x_i's, y_i's, and z_i's, where $1 \le i \le k$, are colored with 1, 2 and 3, respectively). We see that $c_3(x'_1) = 3$ while $c_3(z'_k) = 1$, which already implies that $x'_1 \ne z'_k$. By z''_k denote the neighbor of z'_k other than z_k. We also remember from Lemma 3 that $c_3(z''_k) = 3$.

Let c_4 be obtained from c_3 by putting $c_4(x'_1) = c_4(z'_k) = 2$ and swapping colors on the path $x_1z_1x_2z_2 \ldots x_kz_k$. (The latter can be done since z'_k is adjacent to none of $y'_1, \ldots, y'_k$.) It is easy to see that c_4 is an acyclic L-coloring of G^*; a contradiction. □

Corollary 2. *No triplet is incident with more than one edge going to a 2-vertex of G.* □

We now introduce a notion crucial for our proof. Let v be a 2-vertex of G adjacent to v_1 and v_2. A *feeding path* $FP_i(v)$ for v, where $i \in \{1,2\}$, is a garland $\mathcal{G}_k$ with the smallest k such that $x'_1 = v$, $x_1 = v_i$, and at least one of y'_k, z'_k, say z'_k, does not belong to a triplet. Then z'_k is called a *sponsor of* v *along* $FP_i(v)$ and denoted by $S_i(v)$.

For each of the two edges vv_1 and vv_2, we start constructing a feeding path $FP_i(v)$ for v. If, say, v_1 does not belong to a triplet, then $FP_1(v)$ is trivial and consists of just one edge vv_1. Suppose $v_1 \in T_1$, then we put $x_1 = v_1$, $T_1 = \{x_1, y_1, z_1\}$. Now if at least one of y'_1 and z'_1 is not in a certain triplet, then this vertex is declared the (first) sponsor of v. Otherwise, using the standard notation for a garland, we consider a triplet T_2 that contains $z'_1 = x_2$, and so on.

By Lemma 4, the feeding path we are constructing cannot come to v along edge vv_2. By Corollary 1, our garland cannot arrive at a triplet already included in it. Since G is finite, we shall eventually finish our feeding path $FP_1(v)$ that starts with edge vv_1 by finding $S_1(v)$. Arguing the same, we get the second feeding path $FP_2(v)$ and sponsor $S_2(v)$ for v.

Corollary 3. *No two feeding paths can intersect except possibly in their sponsors or their initial 2-vertex.*

PROOF. Suppose we have feeding paths $FP_1 = v \dots S(v)$ and $FP_2 = w \dots S(w)$ with a vertex z in common such that z belongs to a triplet. This contradicts Lemma 4. □

In turn, Corollary 3 implies

Corollary 4. *No edge can belong to more than one feeding path.* □

The following lemma has common features with Lemma 4 but is a bit more technical.

Lemma 5. *No 3-cycle has two 3-vertices of G that are sponsors.*

PROOF. Take a garland $\mathcal{G}_k = T_1, \dots, T_k$ such that $d(x'_1) = d(z'_k) = 2$ (it is not excluded that $x'_1 = z'_k$) and suppose that precisely one of its vertices y_q, $1 \le q \le k$, is "spoilt" by having $d(y_q) \ge 4$. Denote this object by $\mathcal{G}_k^q$ (see Fig. 4).

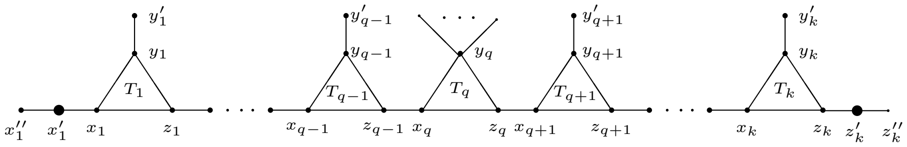

Fig. 4. Quasigarland $\mathcal{G}_k^q$

We have to prove that $\mathcal{G}_k^q$ does not exist. Indeed, we see a feeding path for x'_1 formed by a subgarland $\mathcal{G}_{q-1}$ of $\mathcal{G}_k^q$, with a sponsor x_q, and also a feeding path $FP(z'_k)$ for z'_k, with a sponsor z_q, formed by the sequence $T_k, \dots, T_{q+1}$ of triplets belonging to $\mathcal{G}_k^q$.

Let c be an L-coloring of $G^* - x'_1$. By Lemma 3, we can assume that $c(x''_1) = 1$ and $L(v) = \{1,2,3\}$ for each $v \in \{x'_1\} \cup \mathcal{G}_{q-1}$. Similarly, since $FP(z'_k)$ is a garland, we have $L(v) = \{\alpha, \beta, \gamma\}$ for each $v \in \{z'_k\} \cup T_k \cup \ldots \cup T_{q+1}$.

Let a coloring c_t, where $t \in \{2,3\}$, be obtained from c by putting $c_t(x'_1) = t$ and $c_t(x_i) = 1$, $c_t(y_i) = 5 - t$, $c_t(z_i) = t$ for each $1 \le i \le q-1$. Note that coloring c_t

should create a $(1,t)$-cycle through x'_1, for otherwise we have nothing to prove. In particular, this implies that $c(x_q) = 1$ and $\{c(y_q), c(z_q)\} = \{2, 3\}$. By symmetry, we can assume that $c(y_q) = 2$, $c(z_q) = 3$.

Now the idea is to modify c_3 so as to direct the dangerous $(1,3)$-path along the rest of $\mathcal{G}_k^q$ towards z'_k and there to do a final recoloring that could prevent us from any bicolored cycles in G^*.

It follows from c_3 that $c(x_{q+1}) = 1$ if $q < k$ or $c(z'_k) = 1$ if $q = k$. Hence, $\{1, 3\} \subseteq \{\alpha, \beta, \gamma\}$ in both cases. So, for each $q + 1 \le i \le k$ we have $\{c(x_i), c(y_i), c(z_i)\} = \{1, \beta, 3\}$.

CASE 1. $c(y_{q+1}) = \ldots = c(y_k) = \beta$. Clearly, we are done unless $c(z'_k) = 1$ and $c(z''_k) = 3$. Now if $L(x_q) \neq \{1, 2, 3\}$, then it suffices to modify c_3 to an acyclic L-coloring c'_3 by recoloring x_q with a color from $L(x_q) \setminus \{1, 2, 3\}$. By symmetry, we can assume $L(x_q) = L(z_q) = \{1, 2, 3\}$. In this case we modify c_3 as follows: $c'_3(x_i) = 3$, $c'_3(z_i) = 1$, for $1 \le i \le k$, and, of course, $c'_3(x'_1) = 2$, $c'_3(z'_k) = \beta$.

CASE 2. $c(y_{q+1}) = \ldots = c(y_r) = \beta$, where $r < k$, while $c(y_{r+1}) = 3$ (see Fig. 5). Clearly, we are done unless $c(y'_{r+1}) = 1$. If so, our proof splits. Note that $c(z_{r+1}) = \beta$. By z^*_{r+1} denote vertex x_{r+2} if $r + 2 \le k$ or z'_k otherwise.

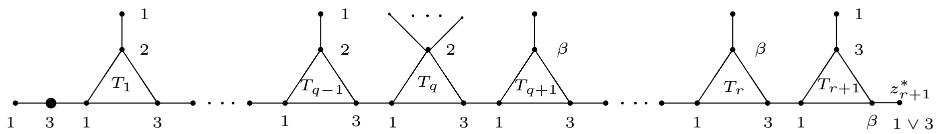

Fig. 5. Coloring c_3 of $\mathcal{G}_k^q$ in Case 2

Subcase 2.1. $c(z^*_{r+1}) = 1$. Here, we recolor y_{r+1} with β and z_{r+1} with 3. If $r + 1 < k$ then we repeat Case 2 with bigger r. If $r + 1 = k$, we go to Case 1.

Subcase 2.2. $c(z^*_{r+1}) = 3$ (see Fig. 6). Here, we obtain an acyclic L-coloring of G by recoloring y_{r+1} with β, all z_i's and x_i's for $1 \le i \le r + 1$ with 1 and 3, respectively, followed by recoloring x'_1 with 2. □

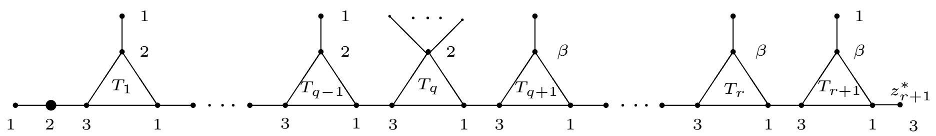

Fig. 6. Coloring c'_3 in Subcase 2.2

2.2. **Completing the proof of Theorem 2.** We discharge the vertices of G as follows:

R1: Each 2-vertex v gets charge 1 along each feeding path started at v.

R2: Every 3-face f gets from each incident vertex v the following charge:
(i) 2 if $d(v) = 3$ and is a sponsor,
(ii) 3 if $d(v) = 3$ and v is not a sponsor,
(iii) 4 if $d(v) \ge 4$.

Finally, we discharge the cut-vertex w^+ of G^+ as follows:

R3: w^+ gives charge 1 along each feeding path started at w^+ and gives charge 4 to every incident 3-face of G^+.

Now check that $ch^*(x) \geq 0$ for every $x \in V(G^+) \cup F(G^+)$.

CASE 1. $f \in F(G^+)$. If $r(f) \geq 12$ then $ch^*(f) = ch(f) = r(f) - 12 \geq 0$. Suppose that $f = xyz$, so $ch(f) = r(f) - 12 = -9$. We have to check that f gets at least 9 from x, y, and z by R2 and R3. Note that at most one of x, y, and z is a sponsor of degree 3 different from w^+ due to Lemma 5, thus giving as little as 2 to f. If none of these is such a sponsor, then $ch^*(f) \geq -9 + 3 \times 3 = 0$. Otherwise, f is not a triplet since no sponsor belongs to a triplet by definition, so f gets 4 from at least one of its incident vertices, and we have $ch^*(f) \geq -9 + 2 + 3 + 4 = 0$.

CASE 2. $v \in V(G)$. If $d(v) = 2$ then $ch^*(v) = -2 + 2 \times 1 = 0$ by R1. Recall that if $d(v) \geq 3$ and v is adjacent to a vertex w such that edge vw does not belong to a 3-face, then v sends at most 1 along edge vw by R1, since feeding paths do not split due to Corollary 4. Also note that v is not incident with two consecutive 3-faces since G has no 4-cycles.

Suppose that $d(v) = 3$; if v does not belong to a 3-face then $ch^*(v) \geq ch(v) - 3 \times 1 = 0$. Otherwise, v either gives 2 to its incident 3-face by R2(i) and sends at most 1 along its non-triangular edge by R1 (if v is a sponsor), or gives 3 by R2(ii), so $ch^*(v) \geq 0$ in both cases.

Finally, suppose that $d(v) = d \geq 4$; then $ch(v) = 5d - 12 \geq 8$. Let t be the number of 3-faces incident with v; we know that $t \leq \lfloor \frac{d}{2} \rfloor$. This implies by R1 and R2(iii) that $ch^*(v) \geq 5d - 12 - 4t - (d - 2t) \times 1 = 4d - 12 - 2t \geq 3(d - 4) \geq 0$.

CASE 3. $ch^*(w^+) \geq 5d + 11 - 4t - (d - 2t) \times 1 = 4d + 11 - 2t \geq 3d + 11 > 0$.

So, after discharging according to rules R1–R3 the charge of each vertex and face of G^+ becomes non-negative, contrary to (1).

This completes the proof of Theorem 2.

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