L. Meykhanadzhyan, S. Matyushenko, D. Pyatkina, R. Razumchik, Совместное стационарное распределение числа заявок в системе с двумя очередями конечной емкости и общим входящим потоком, *Информ. и её примен.*, 2017, том 11, выпуск 3, 106–112

DOI: 10.14357/19922264170312

INFORMATICS AND APPLICATIONS 2017. Vol. 11. Iss. 3. P. 106–112

REVISITING JOINT STATIONARY DISTRIBUTION IN TWO FINITE CAPACITY QUEUES OPERATING IN PARALLEL

L. Meykhanadzhyan[1], S. Matyushenko[2], D. Pyatkina[3], and R. Razumchik[4]

Abstract: The paper revisits the problem of the computation of the joint stationary probability distribution p_{ij} in a queueing system consisting of two single-server queues, each of capacity $N \geq 3$, operating in parallel, and a single Poisson flow. Upon each arrival instant, one customer is put simultaneously into each system. When a customer sees a full system, it is lost. The service times are exponentially distributed with different parameters. Using the approach based on generating functions, the authors obtain a new system of equations of a smaller size than the size of the original system of equilibrium equations ($3N-2$ compared to $(N+1)^2$). Given the solution of the new system, the whole joint stationary distribution can be computed recursively. The new system gives some insights into the interdependence of p_{ij} and p_{nm}. If relations between $p_{i-1,N}$ and $p_{i,N}$ for $i = 3, 5, 7, \ldots$ are known, then the blocking probability can be computed recursively. Using the known results for the asymptotic behavior of p_{ij} as $i, j \to \infty$, the authors illustrate this idea by a simple numerical example.

Keywords: two queues; generating function; stationary distribution; paired customers

DOI: 10.14357/19922264170312

1 Introduction

The system with two single-server queues (both limited and unlimited capacity cases) operating in parallel has received significant attention in the literature due its potential application in real-life scenarios (for example, packet switches, packet radio networks, parallel processing systems, inventory control of database systems, etc.). Further, it is assumed that the system consists of two queues (say, queue 1 and queue 2) each with a single server and there is a single Poisson flow of customers arriving at it. Each customer upon arrival is instantly duplicated: one customer goes to queue 1 and the other goes to queue 2. Both queues are working independently, service times follow exponential distribution with different parameters, and the service discipline in a queue is either FCFS (first-come-first-served), LCFS (last-come-first-served), or Random. Despite the simplicity of the structure, even under such markovian assumptions, the system turned out to be notoriously hard to analyze.

A big list of publications on the topic is given in [1], where the authors give an overview of functional equations (and solution approaches), which arise in the analysis of such systems with infinite capacity queues. References to the application related papers are also given. Among the pioneer works in the area, papers [2–5] are worth noticing.

In this paper, the authors revisit the problem of the computation of the joint stationary distribution in the case, when both queues have finite capacity. Under the exponential assumptions (and given additional dedicated Poisson flows to each queue), the matrix algorithm has been proposed already in [3]. Some further considerations, including the study of correlation between the queues' sizes were continued in [14]. In general, the cases, when both of queues are on finite capacity or one of the queues is (see, for example, [6]), have received less attention in the literature. This is presumably due to the fact that in those cases in order to obtain the joint stationary distribution, one can use widely-adopted general techniques: folding algorithm, linear level reduction or block-gaussian elimination algorithms (see, for example, [7, 8]).

Our motivation for revisiting this problem comes from the papers [9–13], where the generating function technique (which utilizes some properties of special functions (Chebyshev and Gegenbauer polynomials)) was applied to the systems with two finite-capacity queues and allowed one to derive new relations for the recursive computation of the joint stationary distribution.

[1]School No. 281 of Moscow, 7 Raduzhnaya Str. Moscow 129344, Russian Federation; lameykhanadzhyan@gmail.com

[2]Peoples' Friendship University of Russia (RUDN University), 6 Miklukho-Maklaya Str., Moscow 117198, Russian Federation; matyushenko_si@rudn.university

[3]Peoples' Friendship University of Russia (RUDN University), 6 Miklukho-Maklaya Str., Moscow 117198, Russian Federation; pyatkina_da@rudn.university

[4]Institute of Informatics Problems, Federal Research Center "Computer Science and Control" of the Russian Academy of Sciences; 44-2 Vavilova Str., Moscow 119333, Russian Federation; Peoples' Friendship University of Russia, 6 Miklukho-Maklaya Str., Moscow 117198, Russian Federation; rrazumchik@ipiran.ru

Having applied the same approach for the system considered here, the present authors found that it does not lead to the recursive solution. Yet, it gives an alternative way to compute the joint stationary distribution. Specifically, it requires the solution of the system of linear algebraic equations of the size $(3N-2)$, when the size of both queues is equal to $N \geq 3$ (the exact solutions for $N = 1$ and 2 are obtained in [14]) and is immediately suitable for exact arithmetics implementation. If the whole joint stationary distribution is not of importance, this approach gives the straight way to calculate the blocking probability and new insights into the dependencies between the joint probabilities p_{ij}, which prevent the recursive solution.

The paper is structured as follows. In section 2, the description of the system is given and some known results, which are necessary in what follows, are repeated. Section 3 contains the main contribution of the paper. Here, it is shown how new relations for the joint stationary distribution can be obtained (see Eqs. (5)–(12)). The insights into the interdependence between the joint stationary probabilities is discussed in section 4. Section 5 concludes the paper.

2 System Description

The system under consideration consists of two single-server finite capacity queues (queue 1 and queue 2), operating in parallel independently of each other. By suffering a little a lack of generality, let us assume that the capacities of both queues are equal to $N \geq 3$. There is one incoming Poisson flow of rate λ arriving at the system. Upon arrival, each customer is split into two customers: one enters queue 1 and another enters queue 2. Service time of customers in queue i follows exponential distribution with rate μ_i, $i = 1, 2$. Since we are interested here only in the queue size related characteristics, we allow the service discipline in queues to be either FCFS, or LCFS, or Random. We are interested in the case (as in [3]), when a customer always occupies the place in the queue whenever it is not full. This is much different from the case, when the customer checks the queues' sizes before splitting and leaves the system if at least one queue is full.

Denote by p_{ij} stationary probability of the fact that there are i customers in queue 1 and j customers in queue 2. From [3], it follows that the double generating function for p_{ij},

$$P(u,v) = \sum_{i=0}^{N}\sum_{j=0}^{N} u^i v^j p_{ij}\,, \ \ 0 \leq u \leq 1\,,\ 0 \leq v \leq 1\,,$$

has the form:

$$B(u,v)P(u,v) = A(u,v) \qquad (1)$$

where

$$B(u,v) = \lambda u^2 v^2 - u(\lambda v + \mu_1 v + \mu_2 v - \mu_2) + \mu_1 v\,;$$

$$A(u,v) = \mu_1 v\,(u-1)\sum_{j=0}^{N} v^j p_{0j} + \mu_2 u\,(v-1)\sum_{i=0}^{N} u^i p_{i0} + \lambda v^2 u^{N+1}(1-u)\sum_{j=0}^{N} v^j p_{Nj} + \lambda u^2 v^{N+1}(1-v)\sum_{i=0}^{N} u^i p_{iN} + \lambda u^{N+1} v^{N+1}(1-u)(1-v)p_{NN}\,.$$

The quadratic polynomial $B(u,v)$ has two roots:

$$u_{1,2} = u_{1,2}(v) = \Big(v(\lambda+\mu_1+\mu_2) - \mu_2 \mp \sqrt{(v(\lambda+\mu_1+\mu_2)-\mu_2)^2 - 4\lambda\mu_1 v^3}\Big)\Big/(2\lambda v^2)\,.$$

The generating function $P(u,v)$ is the ratio of two polynomial functions. For each value of v, probability generating function $P(u,v)$ is a continuous function of u in the interval $[0,1]$. Then, since the left part in (1) vanishes at points $(u_1(v),v)$, and $(u_2(v),v)$, then the right part must vanish at these points too. In the next section, it will be shown that from this observation, one can obtain the system of linear algebraic equations only for the probabilities $\{p_{0j}, p_{jN}, 0 \leq j \leq N\}$ and $\{p_{j0}, 0 \leq j \leq N-3\}$ which can be solved by any standard numerical method. Once these probabilities are known, the computation of the rest joint stationary probabilities p_{ij} is performed recursively from the system of equilibrium equations.

3 New System of Equations

Both equations $A(u_1(v),v) = 0$ and $A(u_2(v),v) = 0$ share the same unknown quantities. If one expresses term with $\sum_{j=0}^{N} v^j p_{0j}$ from the first equation and put it in the second equation, after collecting common terms, one obtains:

$$\mu_2(v-1)\sum_{i=0}^{N}\left(\frac{u_2^{i+1}-u_1^{i+1}}{u_2-u_1} - u_1u_2\frac{u_2^{i}-u_1^{i}}{u_2-u_1}\right)p_{i0} + \lambda v\,(1-u_1-u_2+u_1u_2)\,\frac{u_2^{N+1}-u_1^{N+1}}{u_2-u_1}\sum_{j=0}^{N} v^{j+1}p_{Nj} + \lambda v^{N+1}(1-v)\sum_{i=0}^{N}\left(\frac{u_2^{i+2}-u_1^{i+2}}{u_2-u_1} - u_1u_2\frac{u_2^{i+1}-u_1^{i+1}}{u_2-u_1}\right)p_{iN}$$

$$+ \lambda v^{N+1}(1 - u_1 - u_2 + u_1 u_2)(1 - v) \times \frac{u_2^{N+1} - u_1^{N+1}}{u_2 - u_1} p_{NN} = 0\,. \quad (2)$$

Instead of cancelling $\sum_{j=0}^N v^j p_{0j}$, let us express the term with p_{NN} from the $A(u_1(v), v) = 0$ and put it into $A(u_2(v), v) = 0$. By doing so, one gets another relation:

$$v\mu_1(1 - u_1 - u_2 + u_1 u_2)\frac{{u_2}^{N+1} - {u_1}^{N+1}}{u_2 - u_1}\sum_{j=0}^N v^j p_{0j} + \mu_2(v-1)\sum_{i=0}^N (u_1 u_2)^{i+1}\left(\frac{{u_2}^{N-i+1} - u_1^{N-i+1}}{u_2 - u_1} - \frac{{u_2}^{N-i} - u_1^{N-i}}{u_2 - u_1}\right) p_{i0} + \lambda v^{N+1}(1 - v)\sum_{i=0}^N (u_1 u_2)^{i+2}\left(\frac{u_2^{N-i} - {u_1}^{N-i}}{u_2 - u_1} - \frac{u_2^{N-i-1} - {u_1}^{N-i-1}}{u_2 - u_1}\right) p_{iN} = 0\,. \quad (3)$$

It is straightforward to see that the roots $u_{1,2}$ admit the following representation:

$$u_1 = \frac{\sqrt{\mu_1}\lambda v}{a}(x)\,; \quad u_2 = \frac{\sqrt{\mu_1}\lambda v}{b}(x)$$

where

$$x = \frac{v(\lambda + \mu_1 + \mu_2) - \mu_2}{v\sqrt{\mu_1 \lambda v}}\,;$$

$$a(x) = \frac{x - \sqrt{x^2 - 4}}{2}\,; \quad b(x) = \frac{x + \sqrt{x^2 - 4}}{2}\,.$$

It can be shown that $|x| > 2$ for all $v \in (0, 1]$. It is well-known that the fraction $(b(x)^m - a(x)^m)/(b(x) - a(x))$ is in fact a polynomial in v for $m \geq 1$. Thus, $(u_2^m - u_1^m)/(u_2 - u_1)$ is a polynomial in v as well. After some tedious algebra (derivation is analogous to the one in [11]), let us find that for $m \geq 1$, the following representation holds:

$$\frac{u_2(v)^m - u_1(v)^m}{u_2(v) - u_1(v)} = \left(\sqrt{\frac{\mu_1}{\lambda}}\right)^{m-1} \sum_{n=\lfloor m/2 \rfloor}^{2(m-1)} v^{-n} a_{m,n} \quad (4)$$

where

$$a_{m,n} = \sum_{j=\max\{0, \lceil n-(m-1) \rceil\}}^{\lfloor (2n-(m-1))/3 \rfloor} d_{m,2n-(m-1)-2j,j}\,, \quad \frac{m-1}{2} \leq n \leq 2(m-1)\,;$$

$$d_{i,m,k} = C_{i-m-1}^{m+1}(0)\binom{m}{k}\left(\frac{\lambda + \mu_1 + \mu_2}{\sqrt{\lambda\mu_1}}\right)^{m-k}\left(-\frac{\mu_2}{\sqrt{\lambda\mu_1}}\right)^k.$$

Here, $\binom{m}{k}$ is the binomial coefficient and $C_n^m(0)$ denotes the value of Gegenbauer polynomial $C_n^m(x)$ at point $x = 0$ (see, for example, [15, p. 175]). Since each fraction $(u_2^m - u_1^m)/(u_2 - u_1)$ is a polynomial in v with real coefficients (defined by (4)) and $u_1 u_2 = \mu_1/\lambda v$, $u_1 + u_2 = ((\lambda + \mu_1 + \mu_2)v - \mu_2)/(\lambda v^2)$, both expressions on the left in (2) and (3) are polynomials in v as well with real coefficients depending on λ, μ_1, μ_2 and certain p_{ij}. Due to the lack of space we omit detailed derivations and just state the final result. From the fact that both polynomials (2) and (3) are equal to zero for $v \in (0, 1]$, it follows that their coefficients are equal to zero. This leads to two systems of linear algebraic equations (one from (2) and the other from (3)) for the stationary probabilities on the boundaries (p_{0j}, p_{i0}, p_{iN}, and p_{Nj}). Careful inspection shows that from these two systems, one can draw one single system of equations of size $3N - 4$ for the probabilities $\{p_{0j}, p_{jN},\ 0 \leq j \leq N\}$ and $\{p_{j0}, 0 \leq j \leq N-3\}$, which can be solved numerically. Specifically, for odd $N \geq 3$, the new system of equations has the form:

$$\sum_{j=0}^N p_{0j} a_{N+1,j+(N-1)/2} + \sum_{j=0}^{(N-1)/2-1} p_{jN}\lambda r^{j+2} b_{N-j,N+(N-1)/2-j} - p_{00}\mu_2 r b_{N+1,(N-1)/2} = 0\,; \quad (5)$$

$$\sum_{j=i}^N p_{0j} a_{N+1,j+(N-1)/2-i} + \sum_{j=0}^{(N-1)/2+(i-1)} p_{jN}\lambda r^{j+2} b_{N-j,N+(N-1)/2-j-i} = 0\,, \quad i = \overline{1, \frac{N-1}{2}}\,; \quad (6)$$

$$\sum_{j=(N+1)/2}^N p_{0j} a_{N+1,j-1} + \sum_{i=0}^{N-1} p_{iN}\lambda r^{i+2} b_{N-i,N-i-1} + p_{NN}\lambda r^{N+1} b_{1,0} = 0\,; \quad (7)$$

$$\sum_{j=N-i}^N p_{0j} a_{N+1,j+i-(N+1)/2} + \sum_{j=0}^{2i+1} p_{jN} r^{j+2} b_{N-j,(N-1)/2+i-j} = 0\,, \quad i = \overline{1, \frac{N-3}{2}}\,; \quad (8)$$

Algorithm 1 Recursive computation of p_{ij}

For $0 \leq i \leq N-2$, $p_{i,N-1} \leftarrow p_{i+1,N}\left(1+\rho_1^{-1}+\rho_2^{-1}\right) - p_{iN} - \rho_1^{-1}p_{i+2,N}$
For $1 \leq j \leq N-2$, $p_{1,j} \leftarrow p_{0,j}\left(\rho_1 + \mu_2/\mu_1\right) - p_{0,j+1}\mu_2/\mu_1$
For $2 \leq i \leq N-1$
  For $1 \leq j \leq N-1$
  $p_{i,j} \leftarrow p_{i-1,j}\left(\rho_1 + 1 + \mu_2/\mu_1\right) - \rho_1 p_{i-2,j-1} - p_{i-1,j+1}\mu_2/\mu_1$
$p_{N-2,0} \leftarrow p_{N-3,0}\left(\rho_1+1\right) - p_{N-3,1}\mu_2/\mu_1$
For $1 \leq j \leq N-1$, $p_{N,j} \leftarrow p_{N-1,j}\left(\rho_1 + 1 + \mu_2/\mu_1\right) - \rho_1 p_{N-2,j-1} - p_{N-1,j+1}\mu_2/\mu_1$
For $N-1 \leq i \leq N$, $p_{i,0} \leftarrow p_{i-1,0}\left(\rho_1+1\right) - p_{i-1,1}\mu_2/\mu_1$

$$\sum_{j=0}^{2i+1} p_{jN} r^j d_{j+1,i} = 0\,, \quad i = \overline{0, \frac{N-3}{2}}\,; \qquad (9)$$

$$\sum_{j=1}^{N} p_{jN}\lambda r^{j-1} d_{j+1,(N-1)/2} = 0\,; \qquad (10)$$

$$\sum_{j=0}^{N} p_{0j} a_{N+1,j+N-2-i} + \sum_{j=0}^{i} p_{jN}\lambda r^{j+2} b_{n-j,2N-2-i-j} - \sum_{j=0}^{N-3-2i} p_{j0}\mu_2 r^{j+1} b_{N+1-i,N-2-i-j} = 0\,, \quad i = \overline{0, \frac{N-5}{2}}\,; \qquad (11)$$

$$\sum_{j=0}^{i} p_{0j} a_{N+1,j+2N-i} - \sum_{j=0}^{i} p_{j0}\mu_2 r^{j+1} b_{N+1-j,2N-i-j} = 0\,, \quad i = \overline{1, N-3}\,, \qquad (12)$$

where the following notations are used:

$$r = \sqrt{\frac{\mu_1}{\lambda}}\,;$$

$$a_{ij} = \mu_2\left(\sqrt{\frac{\mu_1}{\lambda}}\right)^2 C_{i,j}(0) - \mu_1 C_{i,j+1}(0)\,;$$

$$b_{ij} = \sqrt{\frac{\mu_1}{\lambda}}\, C_{i,j}(0) - C_{i-1,j}(0)\,;$$

$$d_{ij} = \sqrt{\frac{\mu_1}{\lambda}}\, C_{i,j}(0) - C_{i+1,j+1}(0)\,.$$

System (5)–(12) consists of $3N-4$ equations in $3N-2$ unknowns. Two additional equations follow from the fact that each queue, when considered independently, operates as the standard $M/M/1/N$ queue and, thus,

$$\sum_{j=0}^{N} p_{0j} = \frac{1-\rho_1}{1-\rho_1^{N+1}}\,; \quad p_{\cdot,N} = \sum_{i=0}^{N} p_{iN} = \rho_2^N \frac{1-\rho_2}{1-\rho_2^{N+1}}\,.$$

Here and henceforth, $\rho_i = \lambda/\mu_i$, $i = 1,2$. Once the system (5)–(12), supplemented with these two equations, is solved, all other probabilities p_{ij} can be found recursively (see Algorithm 1).

The relations in Algorithm 1 follow from the system of equilibrium equations for p_{ij} (see, for example, [3, p. 435]). Algorithm 1 is not well suited for the computation of the whole joint stationary distribution p_{ij} because the accuracy of the results heavily depends on the values of initial parameters and sometimes may be low.

4 Relating $p_{i-1,N}$ and $p_{i,N}$

System (5)–(12) gives some insights into the interdependence of p_{ij} and p_{nm}. Specifically, Eqs. (9) and (10) show that it is enough to know the relations between $p_{2,N}$ and $p_{3,N}$, $p_{4,N}$ and $p_{5,N}$, $p_{6,N}$ and $p_{7,N}$, etc. to compute the value of p_{NN}. Indeed, let $p_{i,N} = p_{i-1,N}\alpha_i$. From (9) and (10), for $y_{jN} = p_{jN}/p_{0N}$, one has:

$$y_{1N} = -\frac{d_{1,0}}{r d_{2,0}}\,; \qquad (13)$$

$$y_{2i,N} = -\frac{\sum_{j=0}^{2i-1} y_{jN} r^j d_{j+1,i}}{r^{2i} d_{2i+1,i} + \alpha_{2i+1} r^{2i+1} d_{2i+2,i}}\,, \quad i = \overline{1, \frac{N-3}{2}}\,; \qquad (14)$$

$$y_{N-1,N} = -\frac{\sum_{j=1}^{N-2} y_{jN} r^{j-1} d_{j+1,(N-1)/2}}{r^{N-2} d_{N,(N-1)/2} + \alpha_N r^{N-1} d_{N+1,(N-1)/2}}\,. \qquad (15)$$

From this system and the normalization condition $p_{\cdot,N} = \rho_2^N(1-\rho_2)/(1-\rho_2^{N+1})$, one finds $p_{0N} = p_{\cdot,N}/\sum_{i=0}^{N} y_{iN}$ and $p_{NN} = p_{0N} y_{N,N}$. We are unaware of any general rule for choosing α_i. Yet, some heuristics can be suggested. Without any loss of generality, further on, we consider $\lambda = 1$. From the results in [5], it follows that if $\mu_2 > \mu_1 > 1$, then for large N, one has:

$$p_{iN} \approx p_{i-1,N} \left(\frac{\Phi(x_{i+1}) - \Phi(x_i)}{\Phi(x_i) - \Phi(x_{i-1})} \right)$$

where

$$x_i = \frac{1}{\sqrt{N}} \left(i - \frac{\mu_2 - \mu_1}{\mu_2 - 1} N \right) ;$$

$$\Phi(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \int\limits_{-\infty}^{x} e^{-t^2/(2\sigma^2)}\, dt ;$$

$$\sigma^2 = \frac{\mu_1^2\mu_2 + \mu_1\mu_2^2 + \mu_1 + \mu_1^2 + \mu_2 + \mu_2^2 - 6\mu_1\mu_2}{(\mu_2 - 1)^3} .$$

Thus, the (approximate) value of p_{NN} can be found from (13)–(15) with

$$\alpha_i = \frac{\Phi(x_{i+1}) - \Phi(x_i)}{\Phi(x_i) - \Phi(x_{i-1})} .$$

It is worth noticing that the value $p_{\cdot,N}(1 - \Phi(x_N))$ gives another approximation for p_{NN} if $\mu_2 > \mu_1 > 1$ and can be quite accurate. We can try one's luck and use the same value of α_i in the overloaded case as well (i. e., when the load of at least one queue is 1) with two minor modifications. Firstly, substitute $-\sigma^2$ instead of σ^2 and, secondly, put $\alpha_i \equiv 1$ whenever $-\sigma^2 < 0$ or $\Phi(x_i) \approx \Phi(x_{i-1})$. With these agreements, by using $p_{i,N} = p_{i-1,N}\alpha_i$ in (13)–(15), the value of p_{NN} can be approximated with $2p_{0N}y_{N,N}$. The data in the table give the idea of the quality of the approximation for the case $N = 35$, $\lambda = 1$, and $\mu_1 = 0.01$ ($\rho_1 = 100$).

Exact values of p_{NN} (solution of (5)–(12)) and approximate values of p_{NN} (solution of (13)–(15)). The case of $N = 35$, $\lambda = 1$, and $\mu_1 = 0.01$

μ_2	p_{NN}	
	Exact value	Approximate value
2.5	$1.1310 \cdot 10^{-15}$	$1.1334 \cdot 10^{-15}$
2	$3.6258 \cdot 10^{-12}$	$3.6380 \cdot 10^{-12}$
1.25	$5.1702 \cdot 10^{-5}$	$5.1934 \cdot 10^{-5}$
1.1	0.0027	0,0027
1.01	0.0217	0.0218
0.9	0.1013	0.1016
0.8	0.1989	0.1972

5 Concluding Remarks

In this paper, it has been shown that the joint stationary probability distribution can be computed using the system of equations of the smaller size (than the original one). The idea follows from the fact that in the finite-capacity case, both roots in the denominator of the generating function are the roots of its numerator (on the contrast to the infinite-capacity case). The drawbacks of the utilized method can be seen when computing the whole joint distribution p_{ij}. Here, the widely-adopted Gaussian elimination and matrix-analytic methods are preferable. Yet, when only the blocking probability is of interest, the utilized method leads to the new computational procedure and some insights into the interdependencies between p_{ij} and p_{nm}. Unlike in some other system with two queues of finite-capacity, here the values of p_{ij} do not allow recursive computation, which is, as clearly seen, due simultaneously happening arrivals. Still the utilized method allows further investigations into the new procedures for the approximate computation of p_{ij} as suggested in [13].

Acknowledgments

This work was supported in part by the Russian Foundation for Basic Research (grants 15-07-03007 and 15-07-03406).

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Received July 15, 2017

Contributors

Meykhanadzhyan Lusine A. (b. 1990) — Candidate of Science (PhD) in physics and mathematics, extended education teaching assistant, School No. 281 of Moscow, 7 Raduzhnaya Str., Moscow 129344, Russian Federation; lameykhanadzhyan@gmail.com

Matyushenko Sergey I. (b. 1963) — Candidate of Science (PhD) in physics and mathematics, associate professor, Peoples' Friendship University of Russia (RUDN University), 6 Miklukho-Maklaya Str., Moscow 117198, Russian Federation; matyushenko_si@rudn.university

Pyatkina Daria A. (b. 1968) — Candidate of Science (PhD) in physics and mathematics, associate professor, Peoples' Friendship University of Russia (RUDN University), 6 Miklukho-Maklaya Str., Moscow 117198, Russian Federation; pyatkina_da@rudn.university

Razumchik Rostislav V. (b. 1984) — Candidate of Science (PhD) in physics and mathematics, leading scientist, Institute of Informatics Problems, Federal Research Center "Computer Science and Control" of the Russian Academy of Sciences, 44-2 Vavilov Str., Moscow 119333, Russian Federation; associate professor, Peoples' Friendship University of Russia (RUDN University), 6 Miklukho-Maklaya Str., Moscow 117198, Russian Federation; rrazumchik@ipiran.ru

СОВМЕСТНОЕ СТАЦИОНАРНОЕ РАСПРЕДЕЛЕНИЕ ЧИСЛА ЗАЯВОК В СИСТЕМЕ С ДВУМЯ ОЧЕРЕДЯМИ КОНЕЧНОЙ ЕМКОСТИ И ОБЩИМ ВХОДЯЩИМ ПОТОКОМ*

Л. А. Мейханаджян[1], С. И. Матюшенко[2], Д. А. Пяткина[2], Р. В. Разумчик[2,3]

[1] Школа № 281 города Москвы
[2] Российский университет дружбы народов
[3] Институт проблем информатики Федерального исследовательского центра «Информатика и управление» Российской академии наук

Аннотация: Рассматривается система массового обслуживания с входящим пуассоновским потоком и двумя приборами, являющаяся одним из простых вариантов fork-систем. Перед каждым прибором имеется накопитель конечной емкости. При поступлении в систему новой заявки создается ее копия и далее в каждую из очередей поступает по одной заявке. Если в момент поступления заявки накопитель оказывается полностью заполненным, заявка теряется и в систему не возвращается. Времена обслуживания заявок на приборах имеют экспоненциальное распределение с различными параметрами. Хорошо известно, что подобные системы с трудом поддаются аналитическому анализу. В работе предлагается метод нахождения вероятности блокировки, а также совместного стационарного распределения числа заявок в накопителях, основанный на методе производящих функций и использующий некоторые результаты теории специальных функций.

Ключевые слова: система массового обслуживания; fork-система; две очереди; конечная емкость; стационарное распределение

DOI: 10.14357/19922264170312

* Работа частично поддержана грантами РФФИ № 15-07-03007 и № 15-07-03406.

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Поступила в редакцию 15.07.2017