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## ESTIMATION OF DYNAMIC YIELD STRESS BY TAYLOR TEST WITH REDUCED CYLINDRICAL HEAD PART OF SAMPLES

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A simple method is proposed to estimate the dynamic yield stress of materials using modified Taylor tests for high-velocity impact of profiled cylinders with a reduced diameter of the head part. Assuming the uniformity of deformations and stresses in the head part, formulas are derived for estimating the yield stress and strain rate from the change in the length of the reduced head part, as well as the mass of the sample and the impact velocity. This estimation is verified by comparison with the results of numerical calculations by the SPH method based on the dislocation plasticity model parameterized for cold-rolled oxygen-free copper. It is shown that the stopping time of the sample and the strain rate are reproduced with good accuracy, and the shear strength estimate gives an error that increases with the impact velocity. At velocities that do not lead to deformation of a wide part of the sample (up to 90 m/s in the case under consideration), the error increases linearly up to 30%, which can be taken into account by a correction factor. The proposed estimate, taking into account the correction factor, was applied to analyze the results of previous experiments; the obtained values correspond to the literature data on the rate dependence of the shear strength.

**Keywords:** *Taylor impact test, profiled cylinder, SPH, dislocation plasticity, estimation of dynamic yield stress.*

### Introduction

The study of the dynamic properties of materials at various degrees and rates of loading is an urgent task nowadays. The knowledge gained about the material makes it possible to determine the options for its application in industry and in the design of various machines or mechanisms. An important aspect is the construction of correct numerical models that would adequately describe the behavior of real samples under various conditions of dynamic and quasi-static loading. Experimental and theoretical estimation of the dynamic strength of the material, its strain rate at various degrees of loading allows one to determine the loads to which the material under test is subjected, as well as to verify the correctness of the experiments in comparison with other authors, and to confirm the correctness of the numerical model.

The classic Taylor test was proposed back in the 40s of the last century [1–3], it implies an axisymmetric collision of a cylindrical impactor with a rigid anvil at impact velocities of several hundred meters per second. Such a collision makes it possible to obtain strain rates near the impact surface up to  $10^5 \text{ s}^{-1}$ . The disadvantage of this test in terms of interpretation of the results is associated with non-uniform deformation of

the sample. Despite this, in pioneering work, an analytical method was proposed for estimating the dynamic yield stress of a material by knowing changes in the sample geometry. The application of the Taylor test is also relevant at the present time, allowing one to study the dynamic properties of materials [4], check and fit various numerical models of plasticity [5; 6]. In our previous work [7], we presented a series of classical experiments on the collision of a cylinder with a rigid anvil, and also carried out numerical 3D modeling by the SPH method [8] using the dislocation plasticity model [9]. The further development of the Taylor test was to modify the classical scheme of the experiment, such as the symmetrical Taylor test with the collision of two opposite samples [10]. We previously proposed a modification of the classic Taylor test using profiled copper cylinders machined in the head part [11]. This modification makes it possible to achieve large deformations (up to about 1) and strain rates (up to  $10^5 \text{ s}^{-1}$ ) at relatively low impact velocities (up to 130 m/s) due to the concentration of impact energy in a geometrically reduced head part. In addition, the use of smaller cylinders in the head part makes the deformation of the material more uniform and allows us to build a simple estimate of the dynamic yield stress and strain rate, which is the subject of this work.

## 1. Experimental setup and specimens

The scheme of the experiment was proposed in previous works [7; 11], here we consider its basic features for the completeness of the description. For dynamic testing, we use a shock tube installed in the laboratory of general and applied physics of the Chelyabinsk State University. The shock tube has been modified to accelerate metal impactors up to 12 mm in diameter, for this we placed a polypropylene tube inside the shock tube that can withstand pressures up to 10 bar. The length of the polypropylene pipe is 2.1 meters, the diameter is 12 mm. A uniform flow of compressed air from the high-pressure chamber to the pumping chamber is provided by a special transitional cone-shaped cuff. The maximum pressure in the pumping chamber is 10 bar, and the pressure adjustment allows us to control the impact velocity. The pressure in the working part of the shock tube is reduced to 0.05 bar with the help of a vacuum pump, which makes it possible to increase the impact velocity of the sample with the obstacle. A polypropylene tape folded in several layers is used as a membrane separating the pumping chamber and the working part of the shock tube. A polished chrome steel anvil is placed at the end of the shock tube. To prevent leakage of pressurized gas into the area in front of the sample, we place the striker in a soft rubber or silicone shell. The sample and shell are lubricated with WD-40 silicone grease, which reduces friction during impactor acceleration, allowing high impact velocities to be achieved. To measure the speed of the sample in front of the anvil, the time-of-flight method is used. A photograph and a diagram of the experimental setup are shown in Fig. 1.

Dynamic test specimens were made of M1T grade oxygen-free copper (similar to C11000 in U.S. and Cu-ETP in Europe). Copper is a widely used material in industry and engineering, which has high electrical and thermal conductivity, good ductility and corrosion resistance. These qualities make it possible to use it in various structures and machines subjected to dynamic loads, from use as membranes, elements of electric machines, to use in rocket engineering and the manufacture of shells, being the basis for the design of some of their types.

The as-received material is cold-rolled cylindrical bars 1 meter long and 8 millimeters in diameter. The rods were cut into cylinders 40 mm long and machined for profiling the head to the following shapes: 1) reduced cylinder 4 mm in diameter and 10 mm long;

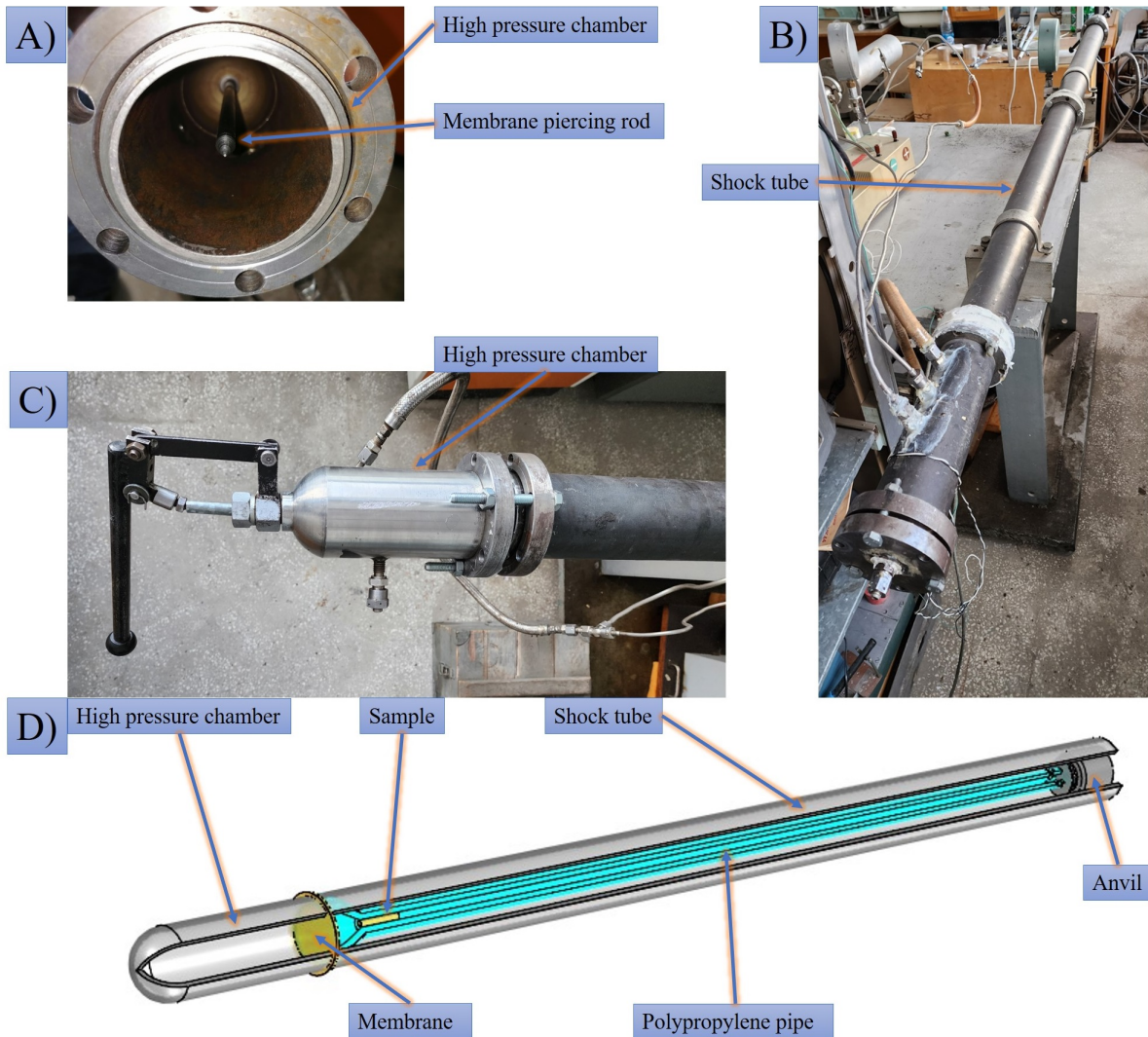


Fig. 1. Photograph and schematic representation of a shock tube modified for dynamic testing: (a) high pressure chamber, inside view; (b) shock tube assembly; (c) high pressure chamber; (d) schematic representation of the shock tube assembly

2) a similar cylinder with a diameter of 3 mm; these sample shapes were first proposed in [11]. The profiling of the cylinders makes it possible to increase the deformation and strain rate in comparison with conventional 40 mm long cylinders at the same impact velocities. Profiling also makes it possible to more adequately estimate the axial strain rate of impactors up to the onset of deformation of the main (not profiled) part of the impactor; for ordinary cylinders, an accurate assessment of the strain rate is difficult to implement without the help of numerical simulation. The mass of the copper sample is about 14.5 and 14 grams for a reduced 4- and 3-mm cylinder, respectively. A schematic representation of the shape of profiled cylinders is shown in Fig. 2.

## 2. Estimation of dynamic strength and strain rate

In this section, we provide simple estimates for important parameters of the dynamic deformation process such as dynamic strength (yield strength of the material), strain rate, and stopping time of the sample. These characteristics are easy to obtain from experimental samples, knowing their spatial dimensions before and after impact, and also taking into account their mass. We use the approximation of uniform deformation and uniform stresses in the head of the reduced cylinder. If there is no deformation of the main 8-mm part, then the kinetic energy of the impactor is completely spent on the

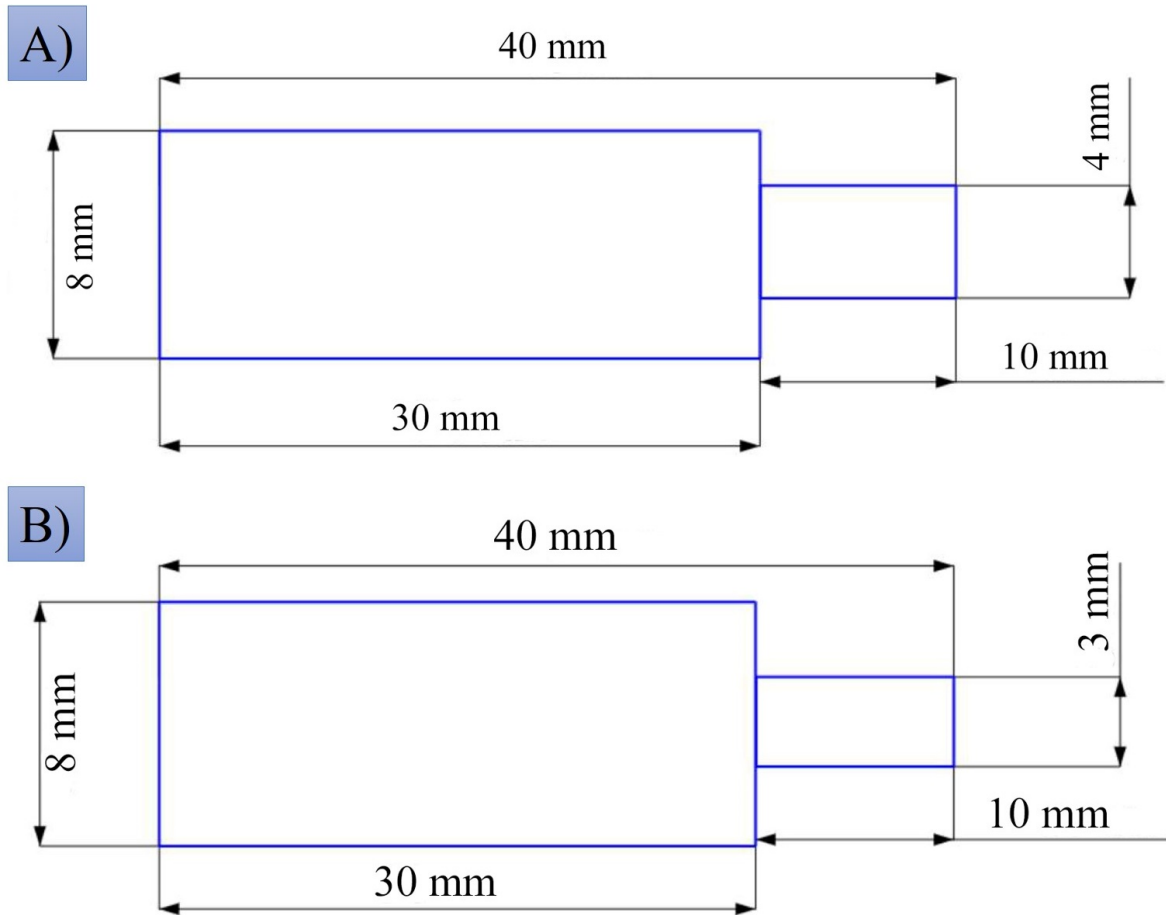


Fig. 2. Schematic representation sample geometry in the case of (a) 4-mm head part and (b) 3-mm head part

plastic deformation of the head part of the impactor. Let us assume that the constant axial stress is equal to the dynamic yield strength or the flow stress and acts in the head part during the deceleration of the projectile, and the radial stress is equal to zero due to lateral unloading. In this case, the shear stress is equal to half of the axial stress  $Y/2$ . A slight shortening  $dL_h$  of the head part with the current length  $L_h$  leads to an increase in the axial strain by  $dL_h/L_h$ , and the work of the axial stress on this deformation is equal to  $Y(dL_h/L_h)V_h$ , where  $V_h$  is the volume of the head part, which is constant during plastic deformation. The total work during braking should be equal to the initial kinetic energy of the impactor, which makes it possible to estimate the dynamic yield strength as

$$Y = mv_0^2 \left[ 2V_h \ln \left( \frac{L_{h0}}{L_{hf}} \right) \right]^{-1}, \quad (1)$$

where  $L_{h0}$  and  $L_{hf}$  are the initial and final length of the head part of the impactor,  $m$  is the mass of the impactor and  $v_0$  is the impact velocity (initial velocity of impactor).

The total true deformation during the impact process is calculated as:

$$\varepsilon_f = \ln(L_{hf}/L_{h0}). \quad (2)$$

The strain rate can be estimated as  $\dot{\varepsilon} \approx \varepsilon_f/t_f$ , where  $t_f$  is the stopping time of the impactor. Let's take a detailer look at the stopping process. Using the law of conservation of energy for intermediate moments of deceleration, we can relate the current length of the head part  $L_h$  with the current velocity of the projectile  $v$  as follows:

$$L_h = L_{h0} \exp \left[ -\frac{m(v_0^2 - v^2)}{2YV} \right]. \quad (3)$$

The sectional area of the head part is  $V/L_h$  taking into account the incompressibility of the material during plastic flow, and the current total force acting on each cross section and decelerating the sample is equal to  $Y (V/L_h)$ . The law of conservation of momentum determines a small decrease in speed  $dV$  in the course of time increment  $dt$ :

$$mdv = -Y (V/L_h) dt. \quad (4)$$

Integration of equation (4) until the impactor stopping, taking into account equation (3), allows us to obtain the following expression for the stopping time:

$$t_f = \sqrt{\frac{\pi}{2}} L_{h0} \sqrt{\frac{m}{YV}} \exp\left(-\frac{mv_0^2}{2YV}\right) \operatorname{erfi}\left(\sqrt{\frac{mv_0^2}{2YV}}\right), \quad (5)$$

where  $\operatorname{erfi}(\cdot)$  denotes the imaginary error function.

### 3. Results and discussion

To estimate the dynamic yield stress of the material, experimental samples from [11] were used. In the case of a 3 mm reduced cylinder, experiments were carried out in the range of velocities of 27–122 m/s; for a reduced 4 mm cylinder, the corresponding range is 19–120.5 m/s. In the range of impact velocities of up to approximately 90 m/s, almost only the shaped head part is subjected to plastic deformation. Such samples are best suited for applying the estimation proposed in Section 2. At higher impact velocities, one can observe deformation of the main 8-mm part of the cylinder, which was not profiled; this deformation worsens the assessment of the dynamic yield stress of the material.

For a theoretical assessment of the dynamic yield stress of the material, its stopping time and axial strain rate, we carried out a numerical modeling of the collision of profiled cylinders. Smoothed particle hydrodynamics (SPH) [8] supplemented by the dislocation plasticity model [9] is used as a numerical method. The collision of classical 8 mm cylinders was implemented in [7]. Cylinder profiling was first applied in [11], and the dislocation kinetics model was parameterized using the statistical Bayesian method. The parameterized numerical model showed good agreement with our own experimental data and with the results on high-speed plate impact experiments from the literature [12]. In this work, modeling was carried out for profiled 3 and 4 mm reduced cylinders, taking into account all experimental impact velocities. In the present study, the number of particles in the system was about 15 thousands, in contrast to [11], in which the number of particles was about 50 thousands. This was done to reduce the calculation time, while reducing the size of the numerical model does not significantly affect the accuracy of modeling, as shown in [11]. Examples of dynamically deformed samples in comparison with numerical ones are shown in Fig. 3.

Equation (1) estimates the dynamic yield stress, and equations (2) and (5) estimate the strain rate and stopping time of the experimental sample. In the case of numerical modelling, we carried out the same assessments in two ways: 1) using the dynamic yield strength, stopping time and strain rate, which were calculated directly in SPH and averaged over the head of the sample; 2) similarly to the experimental estimate, we also carried out the calculation for numerical impactors, using only the dimensions of the sample before and after deformation, as well as its mass and impact velocity.

Fig. 4 shows the results of estimating the strain rate (a,d), stopping time (b,e) and dynamic yield strength (c,f) for the case of reduced 3 and 4 mm cylinders, respectively. For the strain rate, there is good agreement between the calculated data (SPH) and the approximate estimate (Estimation by SPH). At high impact velocities, there is a



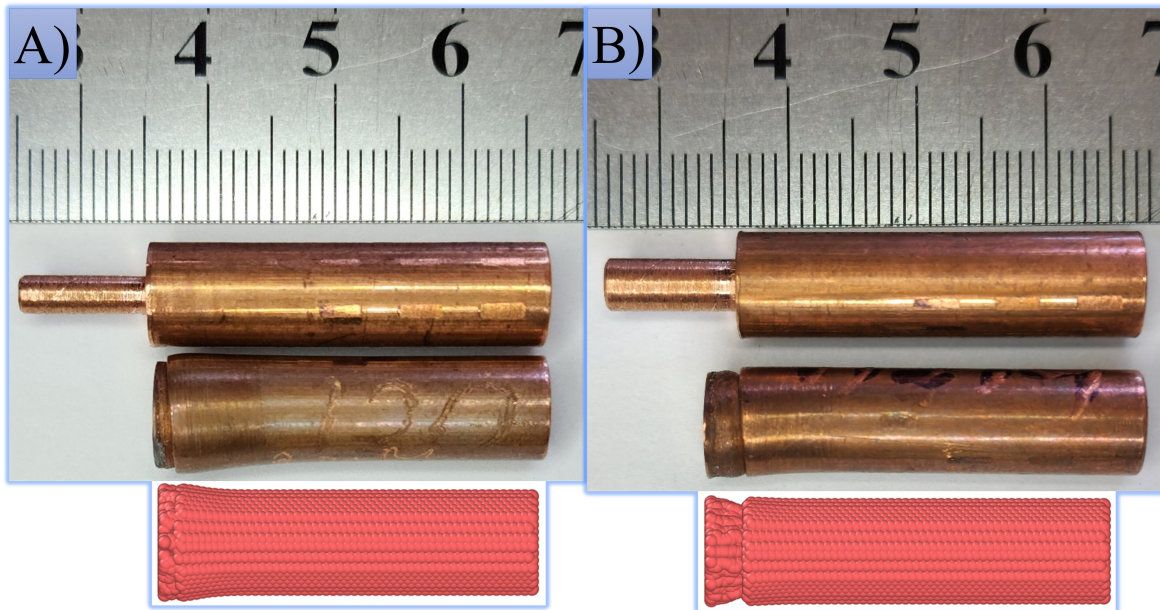


Fig. 3. Dynamically deformed profiled samples, comparison with numerical experiment: (a) profiled 3 mm reduced cylinder, impact velocity is 122 m/s; (b) reduced 4 mm cylinder, impact velocity is 112.4 m/s

discrepancy between the results, which is explained by the deformation of the non-profiled rest part of the cylinder; in this case, the estimate of the strain rate becomes less accurate. For estimation by experimental samples (Estimation by experiments), the maximal axial strain rate is equal to  $2.3 \cdot 10^4 \text{ s}^{-1}$  for 3-mm reduced cylinders and  $1.55 \cdot 10^4 \text{ s}^{-1}$  for 4-mm cylinders.

The estimation of the stopping time of the profiled cylinders shows a good agreement between the estimation from the experimental profiles, the estimation from the SPH profiles, and the calculation of the SPH. A trend towards an increase in the stopping time of impactor at low impact velocities is observed for both shapes of profiled cylinders. In the case of a cylinder reduced to 3 mm, the greatest stopping time is observed at impact velocities of 40–60 m/s, reaching a value of  $165 \mu\text{s}$ , then, with an increase in the impact velocity, the stopping time of the impactor decreases monotonically to values of about 100–120  $\mu\text{s}$ . For a profiled 4 mm cylinder, peak stopping times of about 120  $\mu\text{s}$  occur in the impact velocity range of 50–80 m/s.

In the case of a profiled 4 mm cylinder (Fig. 4, e), when estimating the stopping time of the experimental samples, “fluctuations” of the estimated parameter can be observed. This can be explained by the uneven deformation of the experimental impactors associated with the difference of each of them in the microstructure, the number of internal defects (the experimental material is a cold-rolled drawn bar), as well as the thermal and deformation effects on the samples during their machining. The choice of such a material is due to the fact that, unlike soft hot-rolled copper, its turning is much easier and simpler, but at the same time, we understand that the heterogeneity of its structure can lead to fluctuation of results. This inhomogeneity is best manifested when estimating the dynamic yield stress, which is shown in Fig. 4, c, f.

An analysis of the “SPH” and “Estimation by SPH” curves shows that equation (1) gives an overestimated value of the average yield stress in comparison with the direct SPH calculation. The difference is especially great at impact velocities of more than 90 m/s in both cases of profiled cylinders. This velocity, 90 m/s, is a characteristic one for experimental and numerical samples, because, at higher velocities, plastic deformation of the main 8-mm part of the cylinder begins, which is not taken into account in the

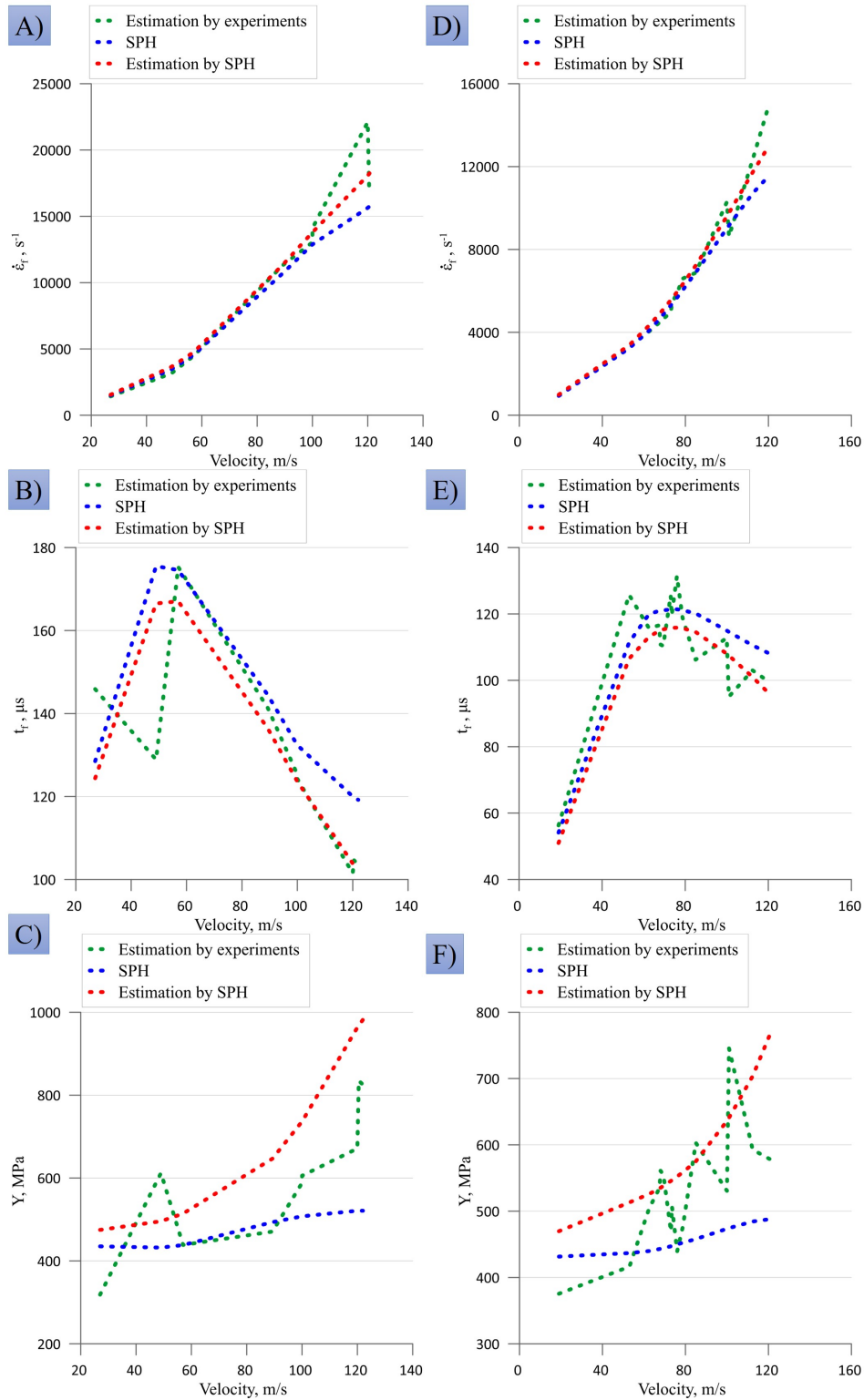


Fig. 4. Estimated parameters versus impact velocity: the strain rate of the profiled part of the reduced cylinders (a,d) with 3-mm and 4-mm head part, respectively; stopping time of the profiled part of the reduced cylinders (b,e) with 3-mm and 4-mm head part, respectively; dynamic yield stress of the reduced cylinders (c,f) with 3-mm and 4-mm head part, respectively

proposed estimation formulas. The deformation of the main part of the cylinders is clearly visible on the samples presented in Fig. 3.

The revealed discrepancy prompted us to calculate a correction factor equal to the ratio of the yield stress according to the estimate (1) by SPH-generated profiles, to a direct SPH calculation. Taking this coefficient into account when processing experimental data will allow us to obtain more accurate experimental estimates of



the dynamic yield stress and partially take into account the influence of the effect of deformation inhomogeneity of the head part of the samples.

To calculate the correction factor, we consider impact velocities up to 90 m/s, at which the plastic deformation of the main part of the 8 mm cylinder is negligibly small. Dependence  $Y_{SPH}/Y_{SPH\ Estim}$  on impact velocity  $v_0$  is shown in Fig. 5. The approximation line is the following expression:

$$Y_{SPH}/Y_{SPH\ Estim} = 0.0029 \cdot v_0 + 1.017. \quad (6)$$

Expression (6) allows us to estimate the value of the coefficient  $Y_{SPH}/Y_{SPH\ Estim}$  for impact velocities up to 90 m/s. It should be emphasized that the estimation of this coefficient at impact velocities of more than 90 m/s does not make much sense due to the beginning deformation of the non-profiled part of the cylinder.

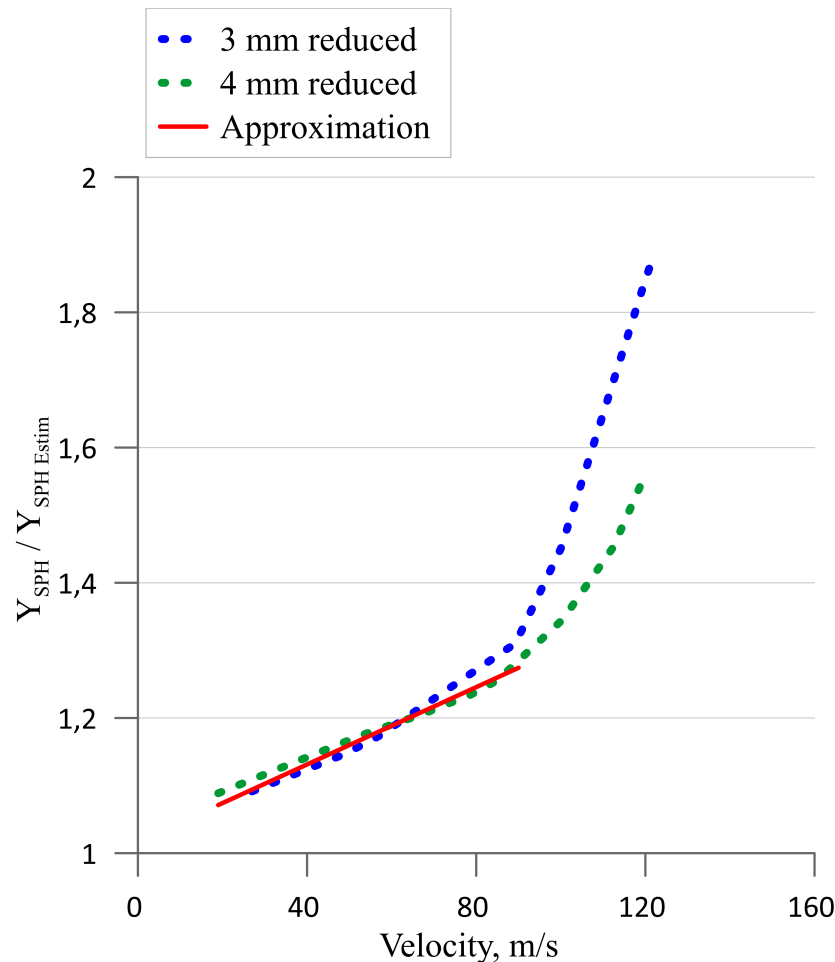


Fig. 5. Ratio  $Y_{SPH}/Y_{SPH\ Estim}$  versus impact velocity for the reduced 3 and 4 mm cylinder. The approximation straight line is the average value of the coefficient for both forms of profiled cylinders

The experiment-based estimate of the dynamic strength depending on the strain rate, obtained with taking into account the correction factor, is shown in Fig. 6 in comparison with the results of other authors. Our data lie in the range of strain rates within  $10^3$ – $10^4$   $s^{-1}$  and exceed by 100–300 MPa the yield strength determined in [13] for small deformations. At the same time, our experimental data correspond to the rate dependence from experiments [14] for a large strain of about 0.5–1. This comparison shows that our results do not contradict the literature data. It should be noted that at a strain rate of the order  $10^3$ – $10^4$   $s^{-1}$  there is a significant increase in the rate sensitivity of the dynamic yield stress.

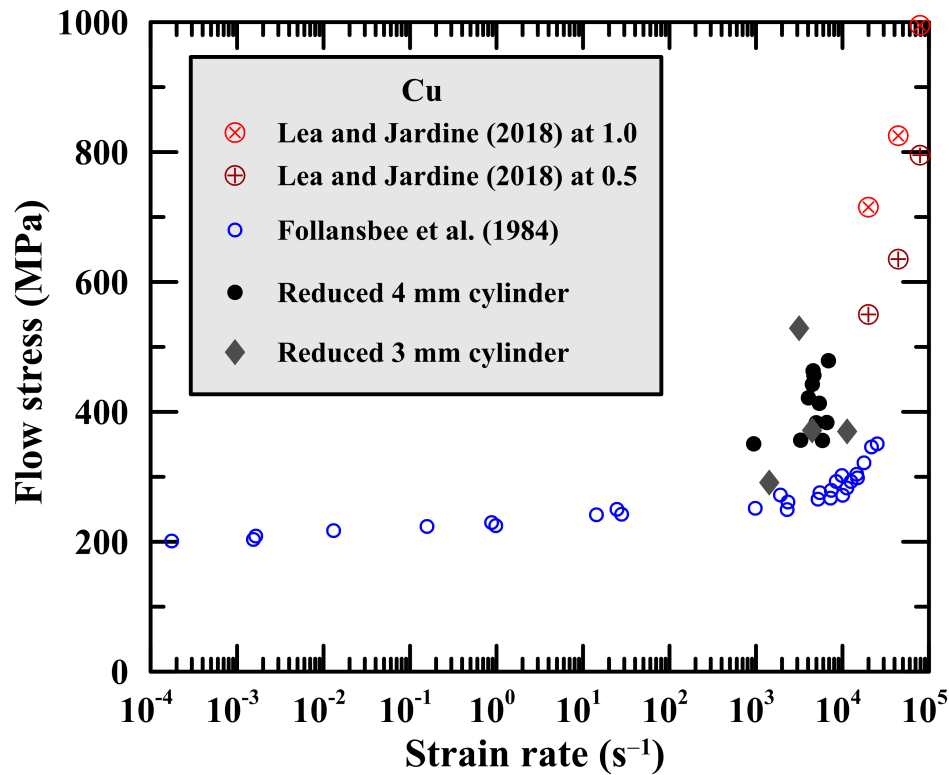


Fig. 6. Dynamic yield stress (flow stress) versus strain rate, our experimental data estimated using equations (1), (2), (5) for the profiled part of 3- and 4-mm reduced cylinder are presented in comparison with experimental data for small deformation (Follansbee et al., 1984) [13] and for true strain 0.5 and 1 (Lea and Jardine, 2018) [14]

## Conclusions

In this paper, we have proposed a new approximate method and estimating the dynamic yield stress and strain rate of copper by using profiled samples. The estimation formulas were verified by comparison with the results of numerical modelling by the SPH method based on the dislocation plasticity model parameterized for cold-rolled oxygen-free copper [11]. It is shown that the stopping time of the sample and the strain rate are reproduced with good accuracy, and the yield stress estimate gives an error that increases with the impact velocity. At speeds that do not lead to deformation of a wide part of the sample (up to 90 m/s in the case under consideration), the error increases linearly up to 30%, which can be taken into account by a correction factor. The proposed estimate, taking into account the correction factor, was applied to analyze the results of previous experiments [11]. It is shown that the data obtained do not contradict the results of other authors on the rate dependence of the dynamic yield strength at high degrees of deformation.

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## ОЦЕНКА ДИНАМИЧЕСКОГО ПРЕДЕЛА ТЕКУЧЕСТИ ПО ТЕСТУ ТЕЙЛОРА С УМЕНЬШЕННОЙ ЦИЛИНДРИЧЕСКОЙ ГОЛОВНОЙ ЧАСТЬЮ ОБРАЗЦОВ

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Предложен простой метод оценки динамической прочности материалов на сдвиг с использованием модифицированных тестов Тейлора на высокоскоростное соударение профилированных цилиндров с головной частью уменьшенного диаметра. В предположении однородности деформаций и напряжений в головной части выведены формулы для оценки сдвиговой прочности и скорости деформации по изменению длины уменьшенной головной части, а также массы образца и скорости соударения. Оценочные формулы проверены путём сравнения с результатами численных расчётов методом SPH на основе модели дислокационной пластичности, параметризованной для холоднокатаной бескислородной меди. Показано, что время остановки образца и скорость деформации воспроизводятся с хорошей точностью, а оценка сдвиговой прочности даёт погрешность, растущую со скоростью соударения. При скоростях, не приводящих к деформации широкой части образца (до 90 м/с в рассматриваемом случае), погрешность линейно растёт до 30 %, что можно учесть поправочным коэффициентом. Предложенная оценка с учётом поправочного коэффициента применена для анализа результатов предыдущих экспериментов; полученные значения соответствуют литературным данным по скоростной зависимости сдвиговой прочности.

**Ключевые слова:** испытание на удар по Тейлору, профилированный цилиндр, гидродинамика сглаженных частиц, дислокационная пластичность, оценка динамического предела текучести.

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