Systemic Interbank Network Risks in Russia

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Interbank network: data description

- Uncollaterized interbank rouble deposits of all maturities in the period from January 11, 2011 till December 30, 2011 are considered.
- Interbank network for N banks is fully characterized by an oriented weighted graph $G^W = (N, W)$, where $W = \{w_{ij}\}$ is an $N \times N$ matrix of $w_{ij} > 0$ of liabilities of the bank i with respect to the bank j.
- By definition the outgoing links correspond to liabilities, the incoming ones - to claims.
- The interbank network graph is scale-free in both in- and out- degrees and is characterized by significant clustering.

Definition of vulnerability

- Solvency coefficient $H1$ as defined by CBRF:

$$H1 = \frac{K}{\sum_i A_i Kp_i + PP + OP + others}$$

- Here
    - K is capital
    - Kp_i - risk coefficients
    - All instruments are divided into 5 groups $i = 1, \cdots 5$ and $Kp_1 = 0$, $Kp_2 = 20$ %, etc. For the interbank market the risk coefficient is 20 %
    - PP - market risk
    - OP - operational risk
    - others - other contributions

Definition of vulnerability

- A default condition is

$$H1 = \frac{K}{\sum_i A_i Kp_i + PP + OP + others} < H1^*$$

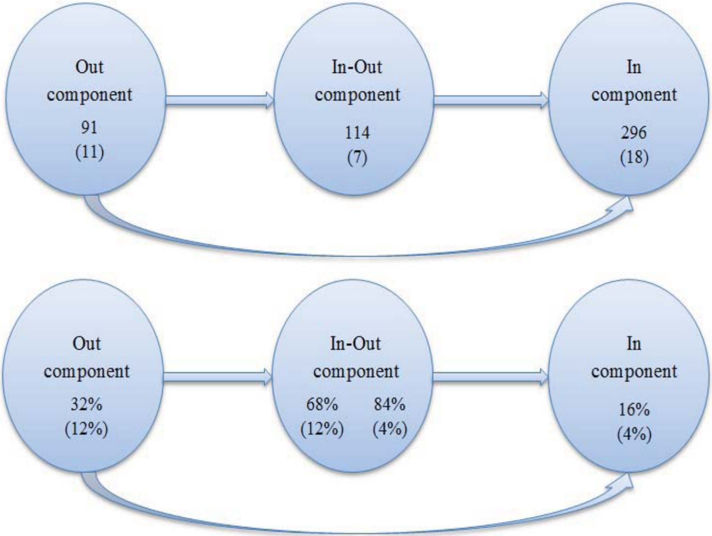
where for banks $H1^* = 10$ %, for others - $H1^* = 12$ %

- Calculation using $H1$:

$$H1 \Rightarrow \frac{K - P}{\sum_i A_i Kp_i + PP + OP + others}$$

where P is a reserve kept for the case when one or several counteragents default.

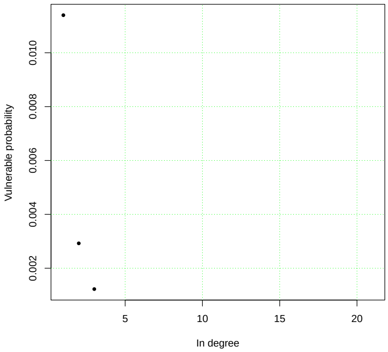
Interbank network: bow-tie structure, nodes and weights



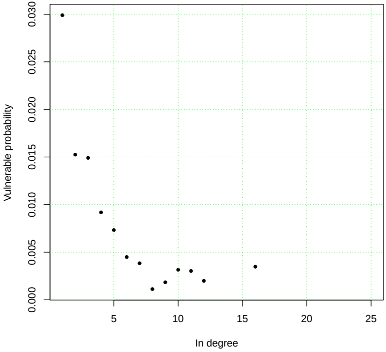
Empirical default distributions

Probability that at least one incoming link is vulnerable:

Out → In-Out



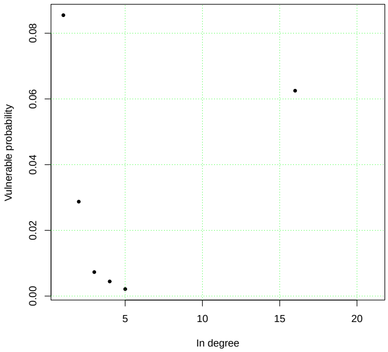
In-Out → In-Out



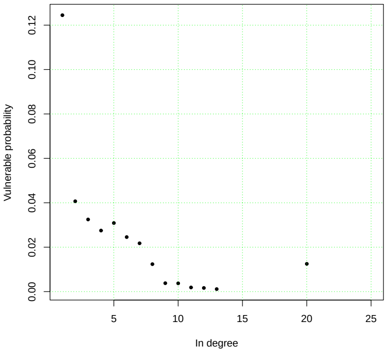
Empirical default distributions

Probability that at least one incoming link is vulnerable:

Out $\longrightarrow$ In

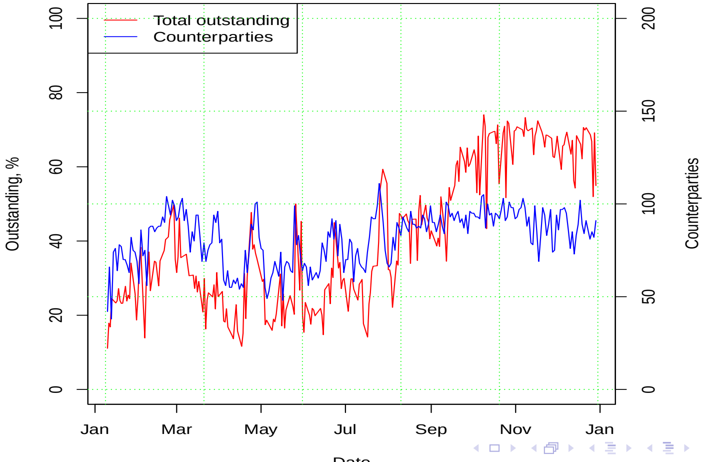


In-Out $\longrightarrow$ In

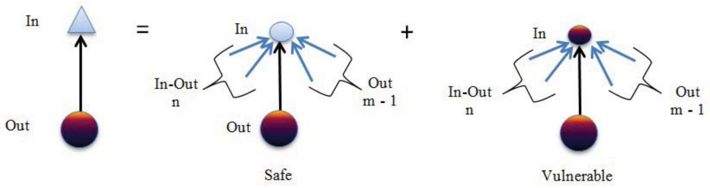


Strongly connected component

- There exists a strongly connected component
- The weight of this component did significantly increase

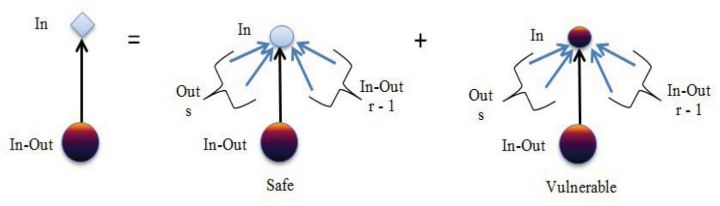


Contagion tree Out → In

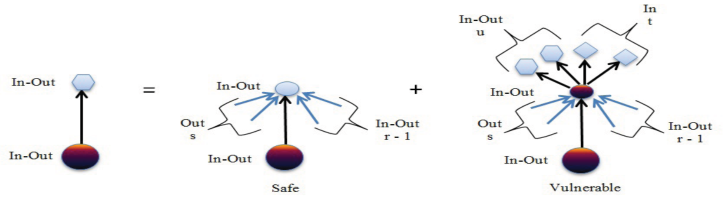


$$L_{i,j}(y) = \sum_{n,m}^{\infty} P_{Out/In}(n,m|i,j) \left[(1 - v_m^{Out/In}) + v_m^{Out/In} y \right]$$

Contagion tree In-Out → In



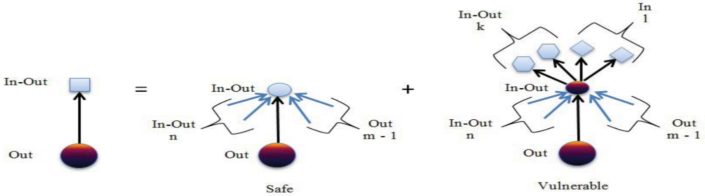
$$N_{k,l}(y) = \sum_{s,r}^{\infty} P_{In-Out/In}(s,r|k,l)\left[(1 - v_r^{In-Out/In}) + v_r^{In-Out/In}y\right]$$



$$M_{k,l}(x, N(y)) = \sum_{u,t,s,r}^{\infty} P_{In-Out/In-Out}(u,t,s,r|k,l)$$

$$* \left[(1 - v_r^{In-Out/In-Out}) + x v_r^{In-Out/In-Out} M_{u,t}^{u}(x, N(y)) N_{u,t}^{t}(y) \right]$$

Contagion tree Out → In-Out → In-Out & In



$$K_{i,j}(x,y) = \sum_{k,l,n,m}^{\infty} P_{Out/In-Out}(k,l,n,m|i,j)$$

$$* \left[(1 - v_m^{Out/In-Out}) + x v_m^{Out/In-Out} [M_{k,l}(x,y)]^k [N_{k,l}(y)]^l \right]$$

Contagion clusters In-Out $\rightarrow$ In-Out & In

- Let $F(x,y)$ be the generation function for the probability for a bank from In-Out being linked with In-Out and In components:

$$F(x,y) = \sum_{i,j=0}^{\infty} p_{ij}^{InOut} x^i y^j$$

- The generation function for default cluster originating in In-Out then reads:

$$\mathcal{G}_{InOut}(x,y) = F(M(x, N(y)), N(y))$$

- The average size of default clusters is given by

$$\left.\frac{d\mathcal{G}_{InOut}(x,x)}{dx}\right|_{x=1} = 1$$

Contagion clusters Out $\rightarrow$ In-Out & In

- Let $G(x,y)$ be the generation function for the probability for a bank from Out being linked with In-Out and In components:

$$G(x,y) = \sum_{i,j=0}^{\infty} p_{ij} x^i y^j$$

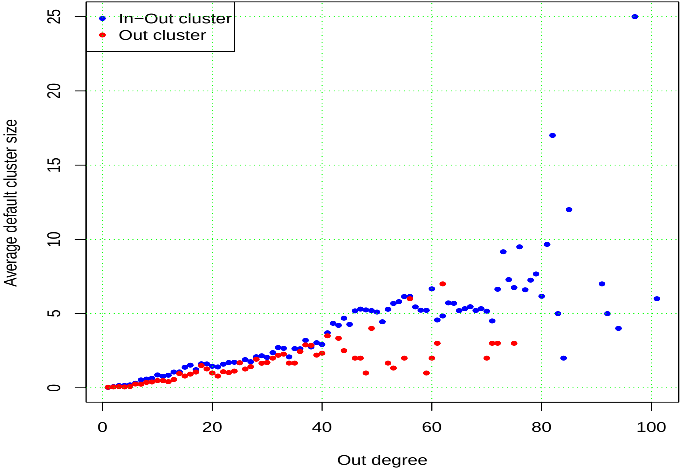
- The generation function for default cluster originating in Out then reads:

$$\mathcal{G}_{Out}(x,y) = G(K(M(x,N(y)),N(y)),L(y))$$

- The average size of default clusters is given by

$$\left.\frac{d\mathcal{G}_{Out}(x,x)}{dx}\right|_{x=1} = 1$$

Simulation: dependence upon out-degree



Simulation: default cluster size distribution