The Pricing Model of Corporate Securities under Cross-Holdings of Equities and Debts

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Plan of Talk

Introduction

Focus

Systemic risk in financial markets and pricing corporate securities

1. Clearing mechanism under cross-holdings (-ownerships) of corporate securities.
2. No-arbitrage prices of corporate securities under cross-holdings

Cross-holdings of debts are not common,

- however the situation of EU countries can be seen as a kind of cross-holdings of debts.

Note:

- The payoff function of corporate securities are not clear if the firms hold corporate securities each other.

Literature review (1)

Stability of the financial system (cross-holdings of debts):

- Eisenberg and Noe (2001):
  - greatest clearing payment vector $\overline{\mathbf{p}}$, least clearing payment vector $\underline{\mathbf{p}}$
  - Under regular condition without default costs, $\overline{\mathbf{p}} = \underline{\mathbf{p}}$.
- Rogers and Veraart (2012) consider default costs. $\overline{\mathbf{p}} \neq \underline{\mathbf{p}}$.

Pricing of securities (cross-holdings of stock and debts):

- Suzuki (2002): cross-holdings of debts and equities.
  - Contraction mapping algorithm
- Fischer (2012): extends with bond seniority structure and derivatives.

We extend Fischer (2012) and consider default costs and introduce early clearing vector $\overline{\mathbf{q}}, \underline{\mathbf{q}}$

Literature review (2)

Structural Models

- Merton (1974):

Firm i

$\tilde{e}_i$ business asset	$p_i^1 = \tilde{e}_i \wedge b_i$ debt
	$p_i^0 = (\tilde{e}_i - b_i)_+$ equity

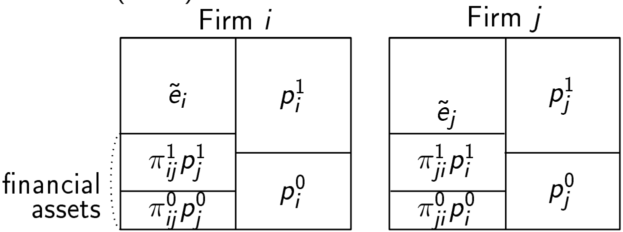
- Pricing formula:

$$\begin{aligned} \text{equity} \quad v_i^0 &= \mathbb{E}^{\mathbb{Q}}[\mathrm{e}^{-rT} \max\{\tilde{e}_i - b_i, 0\}], \\ \text{debt} \quad v_i^1 &= \mathbb{E}^{\mathbb{Q}}[\mathrm{e}^{-rT} \min\{b_i, \tilde{e}_i\}]. \end{aligned}$$

Literature review (3)

Structural Models (cont.)

- Suzuki (2002):



- There is a financial feedback loop among firms under cross-holdings.
- We need to consider the recursion effect on the price of corporate securities.

Our study

What we do in this study:

- Extend Fischer (2012) by introducing default costs.
- Introduce early clearing vector $\underline{\mathbf{q}}, \overline{\mathbf{q}}$

Main results:

- Prove the existence of $\overline{\mathbf{p}}, \underline{\mathbf{p}}$ and $\underline{\mathbf{q}}$ with default costs and debt's seniority structure.
- Show with numerical examples that
  - $\underline{\mathbf{p}}$ is not so serious for the economy.
  - $\underline{\mathbf{q}}$ can capture the credit spreads under financial crisis.
  - Traditional Merton model underestimates the credit risk of debts when we consider cross holding of debts with default costs.

Model Setup

Notations

The following notations are used in this talk:

lower	x, y etc.	scalars
lower and bold	$\mathbf{x}$, $\mathbf{y}$, etc.	vectors
upper and bold	$\mathbf{X}$, $\mathbf{Y}$, etc.	matrices

$$\mathbf{0} = (0, \ldots, 0)^\top, \quad \mathbf{1} = (1, \ldots, 1)^\top, \quad \mathbf{O} = \begin{pmatrix} 0 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & 0 \end{pmatrix}, \quad \mathbf{I} = \begin{pmatrix} 1 & & \mathbf{O} \\ & \ddots & \\ \mathbf{O} & & 1 \end{pmatrix},$$

$$\mathbf{x} \wedge \mathbf{y} = \begin{pmatrix} \min\{x_1, y_1\} \\ \vdots \\ \min\{x_n, y_n\} \end{pmatrix}, \quad \mathbf{x} \vee \mathbf{y} = \begin{pmatrix} \max\{x_1, y_1\} \\ \vdots \\ \max\{x_n, y_n\} \end{pmatrix}, \quad (\mathbf{x})_+ = \mathbf{x} \vee \mathbf{0},$$

Financial markets

- There are totally n firms in the financial market.
- They cross-hold their equities and debts.
- e_i: the business (external) asset of firm i at maturity. Write
$$\mathbf{e} = (e_1, \ldots, e_n)^\top \in \boldsymbol{R}^n_+.$$
- The realization of $\mathbf{e}$ is independent of the structure of cross-holdings.

Debts

- The liability of firm i has a seniority structure with at most m priorities.
- b_i^k: the face value of k-th subordinated debt issued by firm i, $i = 1, \ldots, n$ and $k = 1, \ldots, m$. Write
$$\mathbf{b}^k = (b_1^k, \ldots, b_n^k)^\top \in \boldsymbol{R}_+^n.$$
- Define d_i^k by
$$d_i^k = \sum_{\ell=k+1}^{m} b_i^\ell$$
and
$$\mathbf{d}^k = (d_1^k, \ldots, d_n^k)^\top \in \boldsymbol{R}_+^n.$$

Cross-holding structure and payoffs

- $\pi_{ij}^k \in [0,1]$: proportion of firm i's ownership of b_j^k. Let the $n \times n$ matrix $\mathbf{\Pi}^k = (\pi_{ij}^k)_{i,j=1}^n$, $k = 1, \ldots, m$. Equities are also cross-held with the structure $\mathbf{\Pi}^0 = (\pi_{ij}^0)_{i,j=1}^n$.
- The condition $\mathbf{1}^\top \mathbf{\Pi}^k < \mathbf{1}^\top$ (substochastic matrix) implies the existence of outside investors for k-th debts.
- p_i^k: the payoff of firm i's k-th debt for $k = 1, \ldots, m$, and of firm i's equity for $k = 0$. Write

$$\mathbf{p} = (p_1^0, \ldots, p_n^0, p_1^1, \ldots, p_n^1, \ldots, p_1^m, \ldots, p_n^m)^\top \in \boldsymbol{R}^{(m+1)n}.$$

- We call $\mathbf{p}$ payment vector.

Default costs

- The total asset of firm i before liquidation at maturity, denoted by a_i, is written as

$$a_i(\mathbf{p}) = e_i + \sum_{\ell=0}^{m} \sum_{j=1}^{n} \pi_{ij}^{\ell} p_j^{\ell}.$$

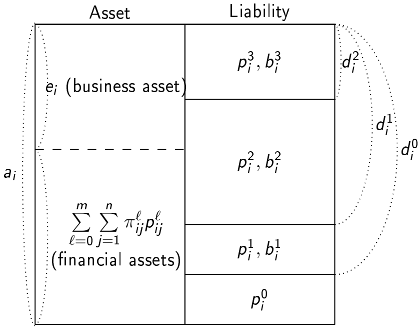
- Default of firm i is determined by the following inequality:

$$\text{firm } i \text{ defaults} \Leftrightarrow a_i(\mathbf{p}) < d_i^0.$$

- If firm i defaults, the payment resource for the debts is reduced to $(1 - \delta_i)a_i$. Here $\delta_i \in [0, 1]$ describes the proportional cost associated with the default. Write $\boldsymbol{\Delta} = \mathrm{diag}[\delta_i]$.

Balance Sheet

In the case $m = 3$:



Definition of clearing payment vector (1)

Definition

We say that $\mathbf{p} \in \boldsymbol{R}_+^{(m+1)n}$ is a clearing payment vector if

$$p_i^0 = \left(a_i(\mathbf{p}) - d_i^0\right)_+$$

and

$$p_i^k = 1_{\{a_i(\mathbf{p}) \geq d_i^0\}} b_i^k + 1_{\{a_i(\mathbf{p}) < d_i^0\}} \left(b_i^k \wedge \left((1-\delta_i)a_i(\mathbf{p}) - d_i^k\right)_+\right)$$

for $i = 1, \ldots, n$ and $k = 1, \ldots, m$.

Definition of clearing payment vector (2)

In matrix form,

$$\mathbf{p}^0 = \left(\mathbf{a}(\mathbf{p}) - \mathbf{d}^0\right)_+$$

and

$$\mathbf{p}^k = (\mathbf{I} - \mathbf{D}(\mathbf{p}))\mathbf{b}^k + \mathbf{D}(\mathbf{p})\left(\mathbf{b}^k \wedge \left((\mathbf{I} - \boldsymbol{\Delta})\mathbf{a}(\mathbf{p}) - \mathbf{d}^k\right)_+\right),$$

where

$$\mathbf{a}(\mathbf{p}) = \mathbf{e} + \sum_{\ell=0}^{m} \boldsymbol{\Pi}^\ell \mathbf{p}^\ell,$$
$$\mathbf{D}(\mathbf{p}) = \mathrm{diag}\left[1_{\{a_i(\mathbf{p})<d_i^0\}}\right].$$

We refer to $\mathbf{D}$ as a default matrix. Remember that

$$\mathbf{p} = \begin{pmatrix} \mathbf{p}^0 \\ \vdots \\ \mathbf{p}^m \end{pmatrix}, \mathbf{p}^\ell = \begin{pmatrix} p_1^\ell \\ \vdots \\ p_n^\ell \end{pmatrix}, \ell = 0, \ldots, m$$

Existence of Clearing Vector

Fixed point problem

- Define the function $\mathbf{f} : \boldsymbol{R}_+^{(m+1)n} \to \boldsymbol{R}_+^{(m+1)n}$ by
$$\mathbf{f}^0(\mathbf{p}) = \left(\mathbf{a}(\mathbf{p}) - \mathbf{d}^0\right)_+,$$
$$\mathbf{f}^k(\mathbf{p}) = (\mathbf{I} - \mathbf{D}(\mathbf{p}))\mathbf{b}^k + \mathbf{D}(\mathbf{p}) \left(\mathbf{b}^k \wedge ((\mathbf{I} - \boldsymbol{\Delta})\mathbf{a}(\mathbf{p}) - \mathbf{d}^k)_+\right)$$
for $k = 1, \ldots, m$.
- The clearing payment vector is expressed as a fixed point $\mathbf{f}(\mathbf{p}) = \mathbf{p}$.
- If $\boldsymbol{\Delta} = \mathbf{O}$, $\mathbf{f}$ is a contraction mapping with respect to l^1-norm (Suzuki, 2002; Fischer, 2012). However, it is not in the case $\boldsymbol{\Delta} \neq \mathbf{O}$.

Existence of clearing payment vector and its Proof (1)

Proposition (Rogers and Veraart; 2012, Nishide and Suzuki; 2013)

Assume that $\mathbf{1}^\top \mathbf{\Pi}^0 < \mathbf{1}$. *Then, there exists a vector* $\mathbf{p} \in \boldsymbol{R}_+^{(m+1)n}$ *such that* $\mathbf{f}(\mathbf{p}) = \mathbf{p}$.

Proof.

Let the vector $\overline{\mathbf{p}}_0^0 \in \boldsymbol{R}_+^n$ *be given by the solution for*

$$\overline{\mathbf{p}}_0^0 = \left(\mathbf{e} + \mathbf{\Pi}^0 \overline{\mathbf{p}}_0^0 + \sum_{\ell=1}^{m} \mathbf{\Pi}^\ell \mathbf{b}^\ell - \mathbf{d}^0\right)_+ .$$

We define the sequence of $(m+1)n$*-dimensional vectors* $\{\overline{\mathbf{p}}_h\}$ *by*

$$\overline{\mathbf{p}}_0 = (\overline{\mathbf{p}}_0^0, \mathbf{b}^1, \dots, \mathbf{b}^m)^\top \text{ and } \overline{\mathbf{p}}_h = \mathbf{f}(\overline{\mathbf{p}}_{h-1}) \text{ for } h \geq 1.$$

Proof of existence (2)

Proof (cont.)

(Sketch)

- *We can show that* $\{\bar{\mathbf{p}}_h\}_{h\geq 0}$ *is monotonically non-increasing.*
- *We can assure* $\{\bar{\mathbf{p}}_h\}$ *is bounded below* ($\bar{\mathbf{p}}_h \geq \mathbf{0}$).
- *Then,* $\{\bar{\mathbf{p}}_h\}$ *has a limit* $\bar{\mathbf{p}}$ *with* $\mathbf{f}(\bar{\mathbf{p}}) = \bar{\mathbf{p}}$.

Remarks of the proof

Remark

The sequence $\{\mathbf{p}_h\}$ is exactly the same as the algorithm proposed by Elsinger (2009). The proposition generalizes his result to the case where the default costs are present.

Remark

Alternatively, we can show the existence of a clearing vector with the sequence $\{\underline{\mathbf{p}}_h\}$ defined by

$$\underline{\mathbf{p}}_0 = \mathbf{0} \text{ and } \underline{\mathbf{p}}_h = \mathbf{f}(\underline{\mathbf{p}}_{h-1}).$$

The proof is completed by noticing that $\{\underline{\mathbf{p}}_h\}$ is a non-decreasing sequence and bounded above ($\underline{\mathbf{p}}_h \leq \overline{\mathbf{p}}_0$).

We denote the limit of $\{\underline{\mathbf{p}}_h\}$ by $\underline{\mathbf{p}}$.

Greatest and least clearing vectors

Proposition

The vector $\overline{\mathbf{p}}$ *is the greatest clearing vector in the sense that*

$$\mathbf{p} \leq \overline{\mathbf{p}}$$

for any $\mathbf{p} \in \boldsymbol{R}_{+}^{(m+1)n}$ *with* $\mathbf{f}(\mathbf{p}) = \mathbf{p}$*. Similarly the vector* $\underline{\mathbf{p}}$ *is the least clearing vector in the sense that*

$$\mathbf{p} \geq \underline{\mathbf{p}}$$

for any clearing vector $\mathbf{p} \in \boldsymbol{R}_{+}^{(m+1)n}$*.*

Notes:

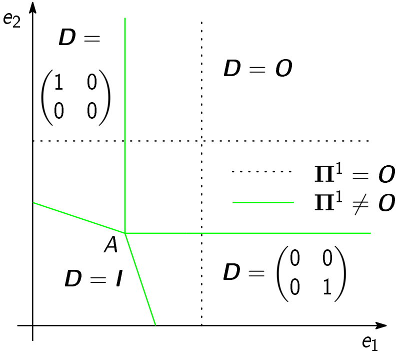
- The proof can be given by contradiction.
- If $\boldsymbol{\Delta} = \mathbf{O}$, then $\underline{\mathbf{p}} = \overline{\mathbf{p}}$

Algorithm for $\overline{\mathbf{p}}$ and $\underline{\mathbf{p}}$

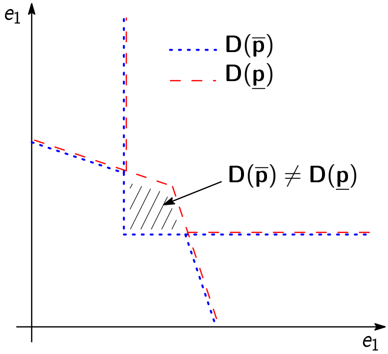
- Define the function $\mathbf{f} : R_+^{(m+1)n} \to R_+^{(m+1)n}$ by
  $$\mathbf{f}^0(\mathbf{p}) = \left(\mathbf{a}(\mathbf{p}) - \mathbf{d}^0\right)_+,$$
  $$\mathbf{f}^k(\mathbf{p}) = (\mathbf{I} - \mathbf{D}(\mathbf{p}))\mathbf{b}^k + \mathbf{D}(\mathbf{p})\left(\mathbf{b}^k \wedge \left((\mathbf{I} - \boldsymbol{\Delta})\mathbf{a}(\mathbf{p}) - \mathbf{d}^k\right)_+\right)$$
  for $k = 1, \ldots, m$.
- Greatest clearing vector $\overline{\mathbf{p}} = \lim_{h\to\infty} \overline{\mathbf{p}}_h$ where
  $$\overline{\mathbf{p}}_0 = (\overline{\mathbf{p}}_0^0, \mathbf{b}^1, \ldots, \mathbf{b}^m)^\top \text{ and } \overline{\mathbf{p}}_h = \mathbf{f}(\overline{\mathbf{p}}_{h-1}).$$
- Least clearing vector $\underline{\mathbf{p}} = \lim_{h\to\infty} \underline{\mathbf{p}}_h$ where
  $$\underline{\mathbf{p}}_0 = \mathbf{0} \text{ and } \underline{\mathbf{p}}_h = \mathbf{f}(\underline{\mathbf{p}}_{h-1}).$$
- We can get $\{\overline{\mathbf{p}}, \mathbf{D}(\overline{\mathbf{p}})\}$ and $\{\underline{\mathbf{p}}, \mathbf{D}(\underline{\mathbf{p}})\}$ for $\exists \mathbf{e}$.

Illustration of D ($n = 2, m = 1, \Pi^0 = O$)

$\underline{\Delta = \mathbf{O}}$ (Suzuki, 2002)



$\underline{\Delta \neq \mathbf{O}}$



- When $\Delta \neq \mathbf{O}$, the region where $\mathbf{D} = \mathbf{I}$ increases, and the region $\overline{\mathbf{p}} \neq \underline{\mathbf{p}}$ comes out.

Motivation of early clearing

- When a large loss is expected in the holding debts, firms must make an allowance for the bad debts even if they are not in default in the market.
- We will examine the situation where every firm values their holding debts as if they are in default.
  - So the asset value of the firm is given by $(1-\delta)a$.
  - If $(1-\delta)a$ is enough to repay the debt, the debt's value is given by its face value.
- In Merton's setting:

$$q^1 = \min\{(1-\delta)a, b\}, \quad q^0 = 1_{\{(1-\delta)a>b\}} \max\{a-b, 0\}$$

Definition of early clearing payment vector (1)

Definition

We say that $\mathbf{q} \in \boldsymbol{R}_{+}^{(m+1)n}$ is an early clearing payment vector if

$$q_i^0 = 1_{\{(1-\delta_i)a_i(\mathbf{q}) \geq d_i^0\}} \left(a_i(\mathbf{q}) - d_i^0\right)_+ \tag{1}$$

and

$$\begin{aligned} q_i^k = & \, 1_{\{(1-\delta_i)a_i(\mathbf{q}) \geq d_i^0\}} b_1^k \\ & + 1_{\{(1-\delta_i)a_i(\mathbf{q}) < d_i^0\}} \left(b_i^k \wedge \left((1-\delta_i)a_i(\mathbf{q}) - d_i^k \right)_+ \right) \end{aligned} \tag{2}$$

for $k = 1, \ldots, m$ and $i = 1, \ldots, n$.

Definition of early clearing payment vector (2)

In matrix form, $\mathbf{q}^0$ and $\mathbf{q}_i^k$ are expressed as

$$\mathbf{q}^0 = \mathbf{S}(\mathbf{q}) \left(\mathbf{a}(\mathbf{q}) - \mathbf{d}^0\right)_+ \tag{3}$$

and

$$\mathbf{q}_i^k = \mathbf{S}(\mathbf{q})\mathbf{b}^k + (\mathbf{I} - \mathbf{S}(\mathbf{q})) \left(\mathbf{b}^k \wedge \left((\mathbf{I} - \mathbf{\Delta})\mathbf{a}(\mathbf{q}) - \mathbf{d}^k\right)_+\right) \tag{4}$$

for $k = 1, \ldots, m$, where

$$\mathbf{S}(\mathbf{q}) = \begin{pmatrix} 1_{\{(1-\delta_1)a_1(\mathbf{q}) \geq d_1^0\}} & & \mathbf{O} \\ & \ddots & \\ \mathbf{O} & & 1_{\{(1-\delta_n)a_n(\mathbf{q}) \geq d_n^0\}} \end{pmatrix}. \tag{5}$$

Existence of early least clearing vector

Proposition

There exists an early clearing vector.

Proof (sketch): Let the function $\mathbf{g} : \boldsymbol{R}_+^{(m+1)n} \to \boldsymbol{R}_+^{(m+1)n}$ be given by

$$\mathbf{g}^0(\mathbf{q}) = \mathbf{S}(\mathbf{q}) \left(\mathbf{a}(\mathbf{q}) - \mathbf{d}^0\right)_+ \tag{6}$$

and

$$\mathbf{g}^k(\mathbf{q}) = \mathbf{S}(\mathbf{q})\mathbf{b}^k + (\mathbf{I} - \mathbf{S}(\mathbf{q})) \left(\mathbf{b}^k \wedge \left((\mathbf{I} - \boldsymbol{\Delta})\mathbf{a}(\mathbf{q}) - \mathbf{d}^k\right)_+\right). \tag{7}$$

Then an early clearing vector is expressed as a fixed point $\mathbf{g}(\mathbf{q}) = \mathbf{q}$.

Algorithm

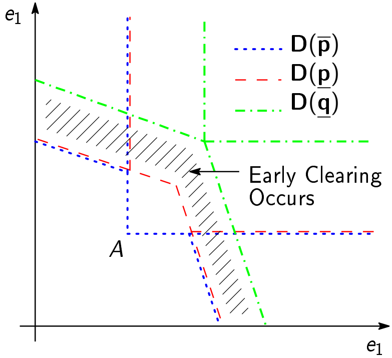
Proposition

The limit $\underline{\mathbf{q}}$ *is the least early clearing vector in the sense that*

$$\underline{\mathbf{q}} \leq \mathbf{q} \tag{8}$$

for any $\mathbf{q}$ *with* $\mathbf{g}(\mathbf{q}) = \mathbf{q}$ *where sequence* $\{\underline{\mathbf{q}}_h\}$ *is defined by* $\underline{\mathbf{q}}_0 = \mathbf{0}$ *and* $\underline{\mathbf{q}}_h = \mathbf{g}(\underline{\mathbf{q}}_{h-1})$.

Illustration of Early Clearing where $n = 2, m = 1, \Pi^0 = O$



Numerical Results

Pricing formula

- Assume that
  - the risk-free rate is a constant r.
  - business assets $\mathbf{e}$ are lognormally distributed.
- Price of debt:

$$v_i^k = \mathbb{E}^{\mathbb{Q}}[\mathrm{e}^{-rT} p_i^k(\mathbf{e})]$$

- Price of equity:

$$v_i^0 = \mathbb{E}^{\mathbb{Q}}[\mathrm{e}^{-rT} p_i^0(\mathbf{e})]$$

- The expectations are taken with respect to the business assets $(e_1, \ldots, e_n)$ under the pricing measure $\mathbb{Q}$.

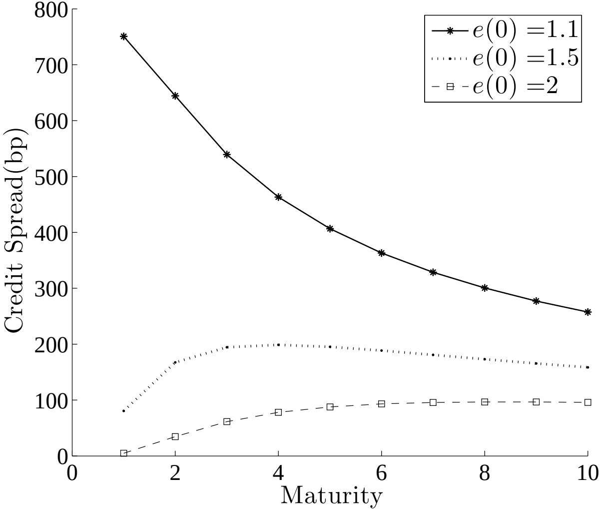
Basic parameters

- Parameters:
  $n = 10, m = 1, \sigma = 0.3, r = 0.05, \delta = 0.4, b_i = 1.0, T = 10$
  $\rho_{ij} = 0.0, \pi^1_{ij} = 0.2, \pi^1_{ii} = 0, \pi^0_{ij} = 0, i, j = 1, \ldots, n$
- Sample pathes $N = 1,000,000$

Notes:

- We suppose $\mathbf{\Pi}^0 = \boldsymbol{O}$. So $\overline{\mathbf{q}} = \underline{\mathbf{q}}$ when $\delta \geq 0$.
- In the numerical example part, we refer to early clearing vector for early least clearing vector.

Credit Spreads by Greatest Clearing Vector under Cross-holding of Debts



Credit Spreads (bp) with Greatest, Least and Early Clearing Vector

	payoffs	1(y)	3	5	7
$e(0) = 2.0$	G^*	4.75599	61.33521	87.76826	95.71702
	L^*	4.76005	61.3808	87.8357	95.78521
	E^*	181.55412	207.03332	185.93967	166.3084
$e(0) = 1.5$	G	80.47351	194.53677	195.15374	180.82115
	L	80.61174	194.84206	195.37659	180.99323
	E	874.91084	474.94717	347.87775	280.36428
$e(0) = 1.1$	G	750.73819	539.26339	406.62969	328.6363
	L	755.51422	540.5624	407.2223	329.00211
	E	2728.66617	971.97699	615.01895	456.95952

G: paid by greatest clearing vector, L: paid by least clearing vector, E: paid by early clearing vector,

The Finding

Early clearing vector can give serious damage to the economy.

Comparison with Merton model (The idea)

- We basically follow Karl and Fischer (2013) and compare our model to Merton (1974) (without cross-holdings).
- Idea
  - Set the initial assets of the two models to be the same.
  - Choose the volatilities and covariance of e_i in Merton's model to match the volatility of a_i in our model.

Simulation Methods

Sequence:

1. Simulate cross-holding model with $\delta = 0$ under basic parameter set.
   - Calculate firm value $x_i = p_i^0 + p_i^1$ and $E[x_i], \Sigma(x_i, x_j)$ for the input values for Merton model.
2. Simulate Merton model under $E[x_i], \Sigma(x_i, x_j)$ if needed.
3. Simulate cross-holding model with $\delta > 0$ under basic parameter set.
4. Simulate Merton model under $E[x_i], \Sigma(x_i, x_j)$ with $\delta > 0$

Purpose:

- To match the distribution of firm value between cross-holding model and Merton model
- To avoid double counting of default loss

VaR and CS of firm 1's Debt

Base Case ($\delta = 0$)

	$\rho = 0, \delta = 0$				$\rho = 0.5, \delta = 0$			
	VaR(1%)		Credit Spread(bp)		VaR(1%)		Credit Spread(bp)	
π^1_{ij}	XOD	Merton	XOD	Merton	XOD	Merton	XOD	Merton
0.30	-0.0381	-0.0747	129.69	118.79	-0.0767	-0.0879	145.39	139.95
0.50	-0.0030	-0.0441	71.38	74.21	-0.0480	-0.0578	91.47	93.87
0.70	0.0244	-0.0165	23.00	40.01	-0.0108	-0.0248	36.41	49.50

Greatest Clearing Vector

	$\rho = 0.0, \delta = 0.4$				$\rho = 0.5, \delta = 0.4$			
	VaR(1%)		Credit Spread(bp)		VaR(1%)		Credit Spread(bp)	
π^1_{ij}	XOD	Merton	XOD	Merton	XOD	Merton	XOD	Merton
0.30	-0.0867	-0.1154	280.85	222.74	-0.1389	-0.1279	299.08	251.61
0.50	-0.0579	-0.0868	222.87	157.82	-0.1291	-0.0995	256.52	187.68
0.70	-0.0384	-0.0616	140.39	99.80	-0.1243	-0.0691	194.65	117.11

Merton model overestimate credit risk with $\delta = 0$

Merton model underestimate credit risk with $\delta > 0, \rho > 0$

Note: $e_i(0) = v_i - \sum_{j=1}^{n} \exp\left(-rT\right)\pi^1_{ij} b_j, v_i = 1.1, i = 1, \ldots, 10$

VaR and CS of firm 1's Debt

Least Clearing Vector

	$\rho = 0.0,\ \delta = 0.4$				$\rho = 0.5,\ \delta = 0.4$			
	VaR(1%)		Credit Spread(bp)		VaR(1%)		Credit Spread(bp)	
π_{ij}^1	XOD	Merton	XOD	Merton	XOD	Merton	XOD	Merton
0.30	-0.0866	-0.1154	281.52	222.74	-0.1388	-0.1279	299.84	251.61
0.50	-0.0579	-0.0868	225.76	157.82	-0.1288	-0.0995	259.87	187.68
0.70	-0.0389	-0.0616	150.47	99.80	-0.1230	-0.0691	207.86	117.11

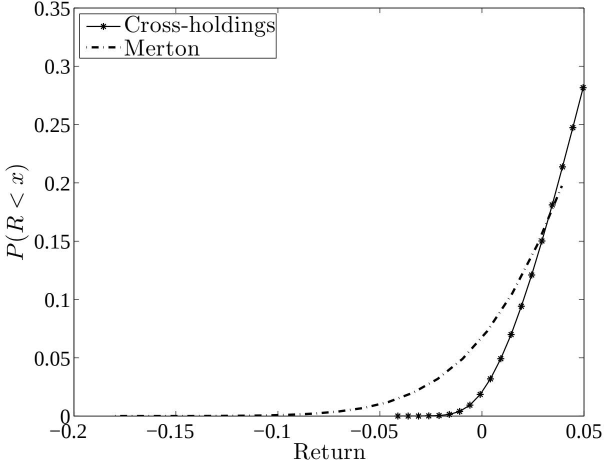
Early Clearing Vector

	$\rho = 0.0,\ \delta = 0.4$				$\rho = 0.5,\ \delta = 0.4$			
	VaR(1%)		Credit Spread(bp)		VaR(1%)		Credit Spread(bp	
π_{ij}^1	XOD	Merton*	XOD	Merton*	XOD	Merton*	XOD	Merton
0.30	-0.0832	-0.1077	370.70	300.03	-0.1314	-0.1203	386.68	326.79
0.50	-0.0542	-0.0786	338.46	240.04	-0.1188	-0.0914	366.45	268.16
0.70	-0.0341	-0.0531	301.47	184.60	-0.1097	-0.0606	342.80	201.56

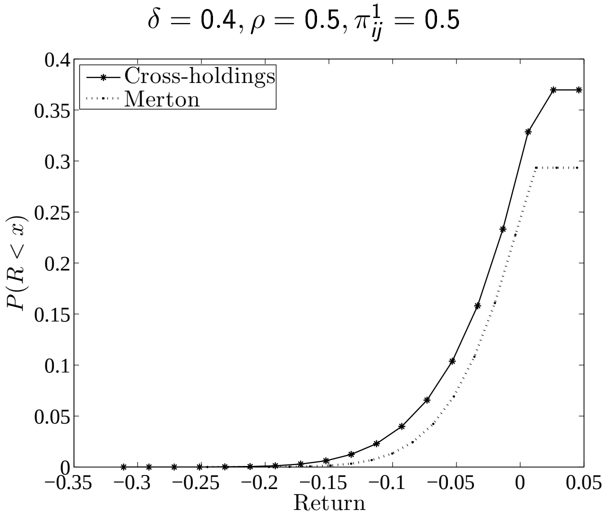
Merton model also underestimate credit risk with $\delta > 0, \rho > 0$, especially in the early clearing

Distribution Functions of Firm 1's Debt Return (Base Case)

$$\delta = 0, \rho = 0, \pi^1_{ij} = 0.5$$

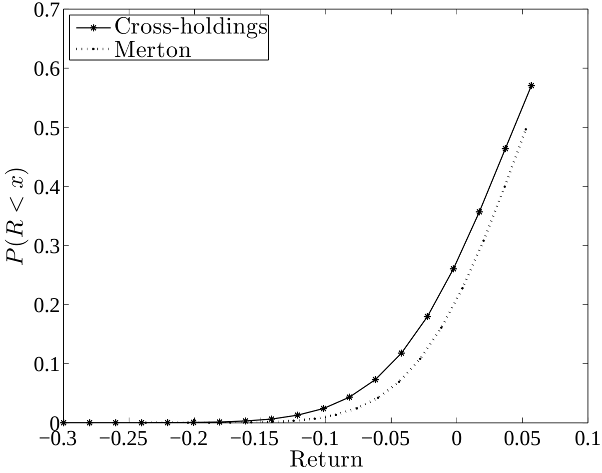


Distribution Functions of Firm 1's Debt Return by Greatest Clearing



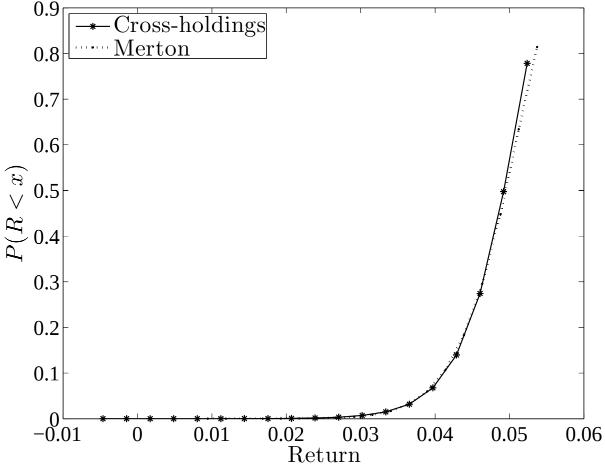
Distribution Functions of Firm 1's Debt Return by Early Clearing

$$\delta = 0.4, \rho = 0.5, \pi^1_{ij} = 0.5$$



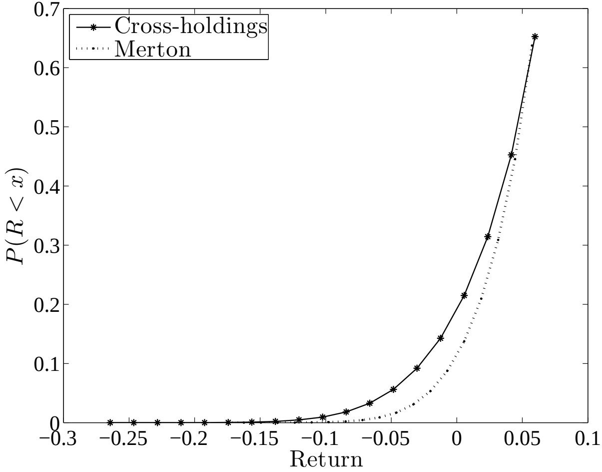
Distribution Function of Bonds Portfolio ($n = 10$) Return (Base Case)

$$\delta = 0.0, \rho = 0.0, \pi^1_{ij} = 0.5$$



Distribution Function of Bonds Portfolio ($n = 10$) Return by Greatest Clearing

$$\delta = 0.4, \rho = 0.5, \pi^1_{ij} = 0.5$$



Conclusion

Conclusion

In this talk, we

- present the pricing model of the corporate securities with cross-ownerships , default costs and bond seniority
- propose an early clearing payment vector $\underline{\mathbf{q}}$ to capture the financial crisis.
- show the existence of $\underline{\mathbf{q}}$.

By numerical example, we

- show $\underline{\mathbf{q}}$ can have serious damage to the economy.
- show Merton model underestimate the credit risk when firms establish cross-holdings of debts when $\delta > 0, \rho > 0$.
- show the underestimation is serious especially in $\underline{\mathbf{q}}$.

Thank you for your attention