Optimal control problem for Sobolev type equations

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The report presents an overview of the work of the Chelyabinsk mathematical school on Sobolev-type equations in the study of optimal control problems for linear Sobolev-type models with Cauchy (Showalter–Sidorov) initial conditions or initial-final conditions [1]. To reveal the non-emptiness of the set of admissible solutions of the control problem, the phase space method is used, which has already proven itself in solving Sobolev-type equations. The method reduces a singular equation to a regular one defined on some subspace of the original space and applies the theory of degenerate (semi)groups of operators to the case of relatively bounded, sectorial or radial operators. Here, mathematical models are reduced to initial (initial-final) problems for an abstract Sobolev type equation. The abstract results are applied to the study of control problems for the Barenblatt–Zheltov–Kochina mathematical model, which describes fluid filtration in a fractured-porous medium, the Hoff model on a graph, which models the dynamics of buckling of an I-beam in a structure, and the Boussinesq-Löve model, which describes longitudinal oscillations in a thin elastic rod, taking into account inertia and under external load or wave propagation in shallow water.

In the works of G.A. Sviridyuk and A.A. Efremov, the problem of optimal control with a quadratic penalty functional for a first-order equation with the Cauchy condition was studied, necessary and sufficient conditions for the existence and uniqueness of the solution were obtained. G.A. Sviridyuk suggested moving from considering the classical solution of the equation to a strong solution of this problem, which made it possible to pose the optimal control problem and use the technique of Hilbert spaces to study it. These studies formed the basis of a number of works by students [2 – 4] and followers of G.A. Sviridyuk on the study of optimal control problems for linear Sobolev type equations based on the theory of degenerate resolving (semi)groups of operators. G.A. Sviridyuk also suggested using a more general Showalter-Sidorov initial condition (initial-final condition), which made it possible to remove the restriction on the set of optimal controls in subsequent works of his students and followers and opened the way to a whole class of problems on this topic.

References:

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