# Algebraic links in the Poincaré sphere and the Alexander polynomials

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The Alexander polynomial in several variables is defined for links in three-dimensional homology spheres, in particular, in the Poincaré sphere: the intersection of the surface $S = \{(z_1, z_2, z_3) \in \mathbb{C}^3 : z_1^5 + z_2^3 + z_3^2 = 0\}$ (the $E_8$ surface singularity) with the 5-dimensional sphere $\mathbb{S}^5_\varepsilon = \{(z_1, z_2, z_3) \in \mathbb{C}^3 : |z_1|^2 + |z_2|^2 + |z_3|^2 = \varepsilon^2\}$. An algebraic link in the Poincaré sphere is the intersection of a germ of a complex analytic curve in $(S, 0)$ with the sphere $\mathbb{S}^5_\varepsilon$ of radius $\varepsilon$ small enough. It is well known that the Alexander polynomial in several variables of an algebraic link in the usual 3-sphere $\mathbb{S}^3_\varepsilon = \{(z_1, z_2) \in \mathbb{C}^2 : |z_1|^2 + |z_2|^2 = \varepsilon^2\}$ (that is of the intersection of a plane curve singularity $(C, 0) \subset (\mathbb{C}^2, 0)$ with $\mathbb{S}^3_\varepsilon$) determines the topological type of the link. We discuss to which extend the Alexander polynomial in several variables of an algebraic link in the Poincaré sphere determines the topology of the link. There exist analytic curves in $(S, 0)$ such that the Alexander polynomials of the corresponding links coincide, but the curves have combinatorially different resolutions. (In this case it is not clear whether or not the links are topologically equivalent.) The dual graph of the minimal resolution of the $E_8$ surface singularity has the standard $E_8$-form. We show that, if the strict transform of a curve in $(S, 0)$ does not

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intersect the component of the exceptional divisor of the minimal resolution corresponding to the end of the longest tail in the corresponding $E_8$-diagram, then its Alexander polynomial determines the combinatorial type of the minimal resolution of the curve and therefore the topology of the corresponding link.

Alexander polynomial of an algebraic link in the Poincaré sphere coincides with the Poincaré series of the filtration defined by the corresponding curve valuations. We show that, under conditions similar for those for curves, the Poincaré series of a collection of divisorial valuations determines the combinatorial type of the minimal resolution of the collection.

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