то задача (1)–(3) имеет единственное s-слабое решение $u(t,x)$ и $u(\cdot) \in$ $\in C([0,T], \widehat{W}_2^{1+s}) \cap C^1([0,T], \widehat{W}_2^s)$. Как и для гиперболических уравнений, возникает аналогичный вопрос, можно ли отказаться от липшицевой непрерывности функций $a(t)$ и $b(t)$, т.е. от условия (6) (см. [2, 3]). Здесь доказана следующая

Теорема. *Пусть выполнены условия* (7)–(9) *и*

$$a(\cdot), b(\cdot) \in \mathrm{BV}[0,T]. \tag{10}$$

Тогда задача (1)–(3) *имеет единственное s-слабое решение* $u(\cdot) \in$ $\in W_2^1(0,T; \widehat{W}_2^{1+s}, \widehat{W}_2^s)$.

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On one ill-posed problem of package guidance*

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The method of program packages is a tool for solution of the guaranteed positional control problems with incomplete information on the initial state of the system. In this talk the application of the method to a linear dynamical system with a linear observed system is considered. An example for which the corresponding problem turns out to be ill-posed is presented.

Let us consider a linear dynamic controlled system described by an ordinary differential equation

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + c(t), \qquad t_0 \le t \le \vartheta. \tag{1}$$

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An open-loop control (program) $u(\cdot)$ is a measurable function on $[t_0, \vartheta]$, $u(t) \in P \subset \mathbb{R}^r$, where P is a convex compact set. The initial state of the system may be not known a priori: $x(t_0) = x_0 \in X_0 \subset \mathbb{R}^n$, where X_0 is a finite known set. The terminal condition $x(\vartheta) \in M \subset \mathbb{R}^n$, where M is a closed and convex set, should hold.

A linear signal $y(t) = Q(t)x(t)$, where $Q(\cdot)$ is left piecewise continuous matrix-function, $Q(t) \in \mathbb{R}^{q \times n}$, $t \in [t_0, \vartheta]$, is observed by the controlling side.

The problem of positional guidance is formulated as follows: based on a given arbitrary $\varepsilon > 0$, choose a closed-loop control strategy with memory such that, whatever the system's initial state $x_0 \in X_0$, the motion $x(\cdot)$ corresponding to the chosen closed-loop strategy and starting at time t_0 from the state x_0 reaches the state $x(\vartheta)$ belonging to the ε-neighbourhood of the target set M at time ϑ.

Definition 1. A homogeneous signal corresponding to an admissible initial state $x_0 \in X_0$ is a function $g(\cdot)$ such that

$$g_{x_0}(t) = Q(t)F(t, t_0)x_0, \qquad t \in [t_0, \vartheta], \quad x_0 \in X_0,$$

where $F(t, s)$, $t, s \in [t_0, \vartheta]$, is the fundamental matrix of the homogeneous system $\dot{x}(t) = A(t)x(t)$ corresponding to (1).

The set of all admissible initial states $x_0 \in X_0$ corresponding to the homogeneous signal $g(\cdot) \in G$ till time point $\tau \in [t_0, \vartheta]$ is given by

$$X_0(\tau | g(\cdot)) = \big\{ x_0 \in X_0 \colon\ g(\cdot)|_{[t_0, \tau]} = g_{x_0}(\cdot)|_{[t_0, \tau]} \big\}.$$

Definition 2. Two arbitrary homogeneous signals $g'(\cdot)$ and $g''(\cdot)$ are initially compatible if

$$\lim_{\zeta \to +0} \big(g'(t_0 + \zeta) - g''(t_0 + \zeta) \big) = 0.$$

Definition 3. A program package is an open-loop control family $(u_{x_0}(\cdot))_{x_0 \in X_0}$ satisfying the non-anticipatory condition: for any homogeneous signal $g(\cdot)$, any time point $\tau \in (t_0, \vartheta]$ and any admissible initial states $x_0', x_0'' \in X_0(\tau | g(\cdot))$, the equality $u_{x_0'}(t) = u_{x_0''}(t)$ holds for almost all $t \in [t_0, \tau]$. The program package $(u_{x_0}(\cdot))_{x_0 \in X_0}$ is guiding if for all $x_0 \in X_0$ it holds that $x(\vartheta | x_0, u_{x_0}(\cdot)) \in M$.

Definition 4. The package guidance problem is solvable if a guiding program package exists.

It was proven that the problem of positional guidance is solvable if and only if the problem of package guidance is solvable [1]; the solvability criterion of the latter and the constructive conditions for identifying the elements of the guiding program package were formulated in [2, 3].

Let us show that the package guidance problem is ill-posed. For this purpose let us consider the following example.

Example 1. Let us consider an arbitrary linear dynamic system (1) and the observed signal $y(t) = Q(t)x(t)$, $t \in [t_0, \vartheta]$, where $Q(t) = (0\ 1)$, $t \in [t_0, \vartheta]$. Let us assume that the program package guidance problem is solvable for the set $X_0 = \{x_0', x_0''\}$, where $x_0' = \binom{x_{01}'}{\delta}$ and $x_0'' = \binom{x_{01}''}{-\delta}$, $\delta > 0$. It is clear that the uniform signals $g_{x_0'}(t) = \delta$, $t \in [t_0, \vartheta]$, and $g_{x_0''} = -\delta$, $t \in [t_0, \vartheta]$, are not initially compatible for any $\delta > 0$; thus, the initial states x_0' and x_0'' belong to different clusters of the cluster position $\mathcal{X}_0(t_0)$ (for the definition of the cluster position, see, e.g., [2]). But if $\delta = 0$, then $g_{x_0'} = g_{x_0''} = 0$, $t \in [t_0, \vartheta]$, and it is impossible to distinguish the initial states x_0' and x_0'', so the solution of the package guidance problem (i.e., the solvability criterion) does not depend continuously on $\delta \to 0$.

One of the possible regularization methods is foreseen in a guidance of the whole δ-vicinity of any initial state $x_0 \in X_0$ for a relatively small $\delta > 0$. The obtained program package will depend on δ; thus, it does not correspond to the original program package; however, taking into account the approximate nature of the initial (positional guidance) problem statement, one can use the methods suggested in [4] to construct the resulting positional strategy satisfying the requirement of ε-guidance.

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