Sirius
International Mathematics Center

051w: Dynamics Days in Sirius

OCTOBER 28–NOVEMBER 1 | 2024

Sirius International Mathematics Center

Dynamics Days in Sirius

October 28–November 1, 2024

Program and Abstracts

Sirius Federal Territory, 2024

Organizers

Anna Chugainova	Steklov Mathematical Insistute of RAS
Andrej Il'ichev	Steklov Mathematical Insistute of RAS
Valery V. Kozlov	Steklov Mathematical Insistute of RAS

Organizations

Sirius International Matematics Center

Steklov Mathematical Institute of Russian Academy of Sciences

Steklov International Mathematical Center

The conference "Dynamics Days in Sirius" is dedicated to the anniversary of the outstanding scientist Academician Dmitry V. Treschev. Talks on the conference are devoted to studying of various problems of qualitative analysis of finite- and infinite-dimensional dynamical systems, as well as the construction of mathematical models for modern analytical and continuum mechanics, including hydrodynamics, gas dynamics and elasticity theory. In particular, there will be discussed a wide class of problems related to the study of various aspects of dynamical systems theory and stability of non-conservative systems, mathematical analysis of wave motions in dispersive and dissipative media, stability of nonlinear wave structures in hydrodynamics, gas dynamics and elasticity theory.

Financial support. The conference is supported by the International Sirius Mathematics Center and by the Ministry of Science and Higher Education of the Russian Federation (the grant to the Steklov International Mathematical Center, agreement no. 075-15-2022-265).

Contents

Conference program

OCTOBER 28, MONDAY

09^{30} — 10^{00} REGISTRATION

10^{00} — 10^{40} Valery Kozlov (online, in Russian)
Remarks on recurrence of Birkhoff sums.

10^{40} — 11^{20} Iskander Taimanov
Central extensions of Lie algebras, dynamical systems, and symplectic nilmanifolds.

11^{20} — 11^{50} Andrej Il'ichev
Motion of particles in the field of nonlinear wave packets and dark solitons in a fluid beneath an ice cover.

COFFEE

12^{30} — 13^{00} Robert MacKay (online)
Hamiltonian theory for confinement of charged particles by magnetic fields.

13^{00} — 13^{30} Luigi Chierchia (online)
Six problems in classical KAM Theory.

13^{30} — 14^{00} Alain Albouy (online)
The two Levi-Civita regularizations

LUNCH

15^{00} — 15^{30} Olga Pochinka
NMS-flows on 4-manifolds.

15^{30} — 15^{50} Marina Barinova
Minimum number of non-wandering points in addition to expanding attractors.

15^{50} — 16^{20} Sergey Bolotin
Another billiard problem.

16^{20} — 16^{50} Vladislav Sidorenko
Quasi-satellites and mini-moons.

18^{00} RECEPTION

OCTOBER 29, TUESDAY

$09^{30} - 10^{10}$ Pavel Plotnikov
Mathematical aspects of brain growth modeling.

$10^{10} - 10^{40}$ Lev Lerman
Spatial dynamics of some 6th-order ODE from the theory of phase transitions.

$10^{40} - 11^{10}$ Tatiana Salnikova
Existence of localized motions near an unstable equilibrium position.

$11^{10} - 11^{30}$ Artem Alexandrov
Dissipative effects in Hamiltonian mean-field model.

COFFEE

$12^{00} - 12^{30}$ Ivan Polekhin
On some geometric ideas in the method of averaging.

$12^{30} - 13^{00}$ Sergey Dobrokhotov
Adiabatic approximation in quantum and classical mechanics, semi-rigid wall billiards and long nonlinear coastal waves.

$13^{00} - 13^{20}$ Alexander Klevin
Nonlinear long standing waves with support bounded by caustics or localized in the vicinity of a two-link trajectory.

$13^{20} - 13^{40}$ Igor Nosikov
Path optimization in inhomogeneous media by a direct approach.

$13^{40} - 14^{00}$ Anna Tsvetkova
Lagrangian manifolds with degenerate fold and applications to the theory of wave beams.

LUNCH

$15^{00} - 15^{40}$ Andrey Mironov
Birkhoff billiards in cones.

$15^{40} - 16^{10}$ Alexander Kuleshov
Application of the Kovacic algorithm to study the problem of the motion of a heavy gyrostat with a fixed point in the integrable Hess case.

$16^{10} - 16^{30}$ Mikhail Garbuz
Dynamic of a spinning heavy disk in an ideal fluid.

$16^{30} - 16^{50}$ Ilya Saraev
Morse-Smale systems on simply connected manifolds.

COFFEE

OCTOBER 30, WEDNESDAY

EXCURSION

OCTOBER 31, THURSDAY

$09^{30} - 10^{10}$ Sergei Kuksin
Long-time behaviour of trajectories for dynamical systems with random forces: non-Markov case.

$10^{10} - 10^{40}$ Alexey Glutsyuk (online)
If a Minkowski Finsler billiard is projective, then it is a standard billiard.

$10^{40} - 11^{10}$ Alexander Bufetov
The Gaussian multiplicative chaos for the sine-process.

$11^{10} - 11^{40}$ Evgeny Zhuzhoma
Codimension one basic sets of Axiom A flows.

COFFEE

$12^{10} - 12^{40}$ Ivan Mamaev
Tensor invariants and conservation laws in mechanics.

$12^{40} - 13^{10}$ Alexander Kilin
Construction of a three-dimensional bifurcation diagram in the problem of a rigid body rolling on a plane.

$13^{10} - 13^{40}$ Andrey Dymov
Rigorous derivation of the wave kinetic equation in subcritical scaling without Feynman diagrams

$13^{40} - 14^{00}$ Alexey Elokhin
Peierls' theory of thermal conductivity and the method of quasisolutions.

NOVEMBER 1, FRIDAY

$10^{00} - 10^{30}$ Anatoly Neishtadt (online)
On probability of capture into 1:1 ground-track resonance while descending towards an asteroid.

$10^{30} - 11^{00}$ Ruzana Polekhina
Admissibility of discontinuities in the solutions of a hyperbolic 2×2 *system of conservation laws.*

$11^{00} - 11^{40}$ Andrei Shafarevich
Semi-classical Asymptotics for Dirac Equations in Abruptly Varying External Fields.

COFFEE

12^{10} — Dmitry Treschev
On quantum Floquet theorem.

Abstracts

28.10
13:30-14:00

The two Levi-Civita regularizations

Alain Albouy

Observatoire de Paris, CNRS, France

Levi-Civita published in 1916, and republished in 1920 and 1924, a proof that the flow of the 3-dimensional 3-body problem may be regularized, that is, replaced by a smooth flow which passes the binary collisions without experiencing any kind of singularity. This proof is reproduced by Siegel in his book (1956). But from 1965, all the authors on the regularization of the binary collisions considered that Levi-Civita only did the 2-dimensional case, and that the 3-dimensional case was first proposed in 1964 by Kustaanheimo, who extended in a sophisticated way the 2-dimensional regularization that Levi-Civita used in 1906. This is extremely surprising. What is then the status of Levi-Civita's result in 1916? He claimed to regularize the flow, without any hypothesis on the value of the energy. If we replace his second regularization by the Kustaanheimo-Stiefel regularization, we cannot regularize the binary collisions with zero energy. If we replace it by Moser's regularization (1970), which indeed Moser relates to Levi-Civita's second regularization (see [J M70] p. 615), we have the same problem. I will propose a very simple improvement of Levi-Civita's second regularization which confirms his argument. The improved regularization is still simpler than the KS regularization.

I wish to thank G.F. Gronchi for informing me of the 3-dimensionality of [T L20].

[J M70] J. Moser, *Regularization of kepler's problem and the averaging method on a manifold*, Communications on Pure and Applied Math. (1970), pp. 609–636.

[T L20] T. Levi-Civita, *Sur la régularisation du problème des trois corps*, Acta Mathematica **42** (1920), pp. 99–144.

Dissipative effects in Hamiltonian mean-field model

29.10
11:10-11:30

Artem Alexandrov

Institute for Information Transmission Problems & Moscow Institute of Physics and Technology & HSE University, Moscow

Hamiltonian mean-field model is the simplest possible realization of many-particle system with long-range interactions. This model has the following Hamiltonian,

$$H_{\text{HMF}} = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \frac{2K}{N} \sum_{i<j}^{N} \cos\left(\phi_j - \phi_i\right), \tag{1}$$

where $\forall i \; \phi_i \in [-\pi, +\pi)$ are phase variables, K is the coupling constant, and N^{-1} factor in the interaction term is needed for the correctly defined $N \to \infty$ limit (so-called *Kac prescription*). It is well-known fact that due to the weak convergence between empirical (discrete) & continuous measures, the HMF model has a well-defined hydrodynamic limit, which is described by the Vlasov equation. In physics HMF model is known as *kinetic* XY-model because of the kinetic energy term in the Hamiltonian. From the physical point of view, XY-model with quenched disorder is of a great interest. This quenched disorder can be modelled by adding into the equations of motion for ϕ_i random quantities ω_i with known distribution function $g(\omega)$.

In addition to quenched disorder, the dissipative processes are also common for physics. The equations of motion for model with disorder and dissipation are given by

$$m\ddot{\phi}_i + \dot{\phi}_i = \omega_i + \frac{2K}{N} \sum_{j=1}^{N} \sin\left(\phi_j - \phi_i\right). \tag{2}$$

In theory of dynamical systems, this model is known as *Kuramoto model with inertia*. All the statement about the weak convergence of measures hold for this model, too. My talk is devoted to generalization of Kuramoto model with inertia for the case with additional stochasticity (dynamical disorder) and for the more complicated structure of couplings, where the interaction is defined by the weighted non-directed graph. The outline of talk consists of

1. Brief review of HMF model
2. Usage of graphons to construct $N \to \infty$ limit
3. Application of Penrose method to analyze stability of incoherent state (uniformly distributed phases)

The concrete set of statements will be adjusted in accordance with timing. The talk is based on papers [CM19a, CM19b, CMM23] and the author's paper [AG24].

[AG24] A. Alexandrov and A. Gorsky, *Penrose method for kuramoto model with inertia and noise*, Chaos, Solitons & Fractals **183** (2024).

[CM19a] H. Chiba and G. S. Medvedev, *The mean field analysis of the Kuramoto model on graphs I. The mean field equation and transition point formulas*, Discrete and Continuous Dynamical Systems - Series A **39**:1 (2019), pp. 131–155.

[CM19b] H. Chiba and G. S. Medvedev, *The mean field analysis of the Kuramoto model on graphs II. Asymptotic stability of the incoherent state, center manifold reduction, and bifurcations*, Discrete and Continuous Dynamical Systems **39**:7 (2019), pp. 3897–3921.

[CMM23] H. Chiba, G. S. Medvedev, and M. S. Mizuhara, *Bifurcations and patterns in the Kuramoto model with inertia*, Journal of Nonlinear Science **33**:5 (2023).

Minimum number of non-wandering points in addition to expanding attractors

28.10
15:30-15:50

Marina Barinova

HSE University

The structure of the non-wandering set of an Ω-stable diffeomorphism closely related to the topology of the ambient manifold and dynamical properties of the system. Moreover, meaningful conclusions can be drawn, even if only part of the non-wandering set is known.

Let $f : M^3 \to M^3$ be an Ω-stable diffeomorphism given on a closed connected 3-manifold M^3. Non-wandering set $NW(f)$ of the diffeomorphism f is a disjoint union of basic sets, each of which is compact, invariant and topologically transitive. If a basic set is a periodic orbit, then it is called trivial, otherwise — non-trivial.

A. Brown proved [A B10], that a non-trivial hyperbolic attractor $\mathcal{A}$ of a 3-diffeomorphism can be only several types:

1. 1- or 2-dimensional expanding attractor, i.e. $\dim \mathcal{A} = \dim W^u_x$, $x \in \mathcal{A}$. A local structure of such attractors is a direct product of a Cantor set and a disk. They can be orientable or non-orientable (in Grines sence).

2. 2- or 3-dimensional Anosov torus. It means that the restriction of the diffeomorphism on the attractor is conjugated to Anosov diffeomorphism given on 2- or 3-torus. In the last case $\mathcal{A}$ coincides with M^3. Anosov torus is always orientable.

We study a class of diffeomorphisms for which all non-trivial basic sets are attractors. Results from paper [BPY24] show that a non-trivial attractor can be either 1-dimensional non-orientable expanding or 2-dimensional expanding (orientable or not) in this case.

Suppose, that each non-trivial attractor is 2-dimensional. We obtained a lower estimates of a number of periodic points outside of non-trivial part of non-wandering set. The estimates are based on a number of connected components of the set $W^2_\Lambda \setminus \Lambda$, where Λ is the union of all non-trivial attractors of the diffeomorphism f. More precisely, the following theorem holds [Bar24].

Theorem 1. *Let $f : M^3 \to M^3$ be an Ω-stable diffeomorphism, given on a closed 3-manifold, Λ be a non-empty set of non-trivial basic sets of f. If Λ consists of 2-dimensional expanding attractors having a* *total of* k_1 *components of the set* $W^s_\Lambda \setminus \Lambda$*, each of which is homeomorphic to* $\mathbb{R}P^2 \times \mathbb{R}$*, and* k_2 *other components, then the number of points in the set* $NW(f) \setminus \Lambda$ *no less then* $\frac{3}{2}k_1 + k_2$ *and this estimate is exact.*

The article was prepared within the framework of the project ”International academic cooperation HSE University”.

[A B10] A. Brown, *Nonexpanding attractors: conjugacy to algebraic models and classification in 3-manifolds*, Journal of Modern Dynamics **4** (2010), pp. 517–548.

[Bar24] M. Barinova, *On isolated periodic points of diffeomorphisms with expanding attractors of codimension 1*, Arxiv. org, 2024.

[BPY24] M. Barinova, O. Pochinka, and E. Yakovlev, *On a structure of non-wandering set of an â¦-stable 3-diffeomorphism possessing a hyperbolic attractor*, Discrete and Continuous Dynamical Systems **44**:1 (2024), pp. 1–17.

Another billiard problem

28.10
15.50-16.20

Sergey Bolotin

Steklov Mathematical Institute of RAS, Moscow

Let (M, g) be a Riemannian manifold, $\Omega \subset M$ a domain with smooth boundary Γ, and ϕ be a smooth function such that $\phi|_\Omega > 0$, $\phi|_\Gamma = 0$, and $d\phi|_\Gamma \neq 0$. We study the geodesic flow of the metric $G = g/\phi$ in Ω. The G-distance from any point of Ω to Γ is finite, so the geodesic flow is incomplete. Regularization of the flow in a neighborhood of Γ establishes a natural reflection law from Γ. This leads to a certain billiard-like problem in Ω. We obtain a normal form for the regularized flow near Γ and for the corresponding billiard map of $T^*\Gamma$. This leads to a version of Lazutkin's theorem [V L72] on the existence of caustics for convex billiards. Our work was motivated by the results of Dobrohotov and Nazaikinkii, see e.g. [SVB13], on the quasi-classical approximation for the wave equation $u_{tt} = \nabla \cdot (\phi \nabla u)$ in Ω degenerating on Γ. The talk is based on the paper [SD24].

This is a joint work with Dmitry Treschev.

[SD24] S. Bolotin and D. Treschev, *Another billiard problem*, Russ. J. Math. Phys. **31** (2024), pp. 50–59.

[SVB13] S. Dobrokhotov, V. Nazaikinskii, and B. Tirozzi, *Two–dimensional wave equation with degeneration on the curvilinear boundary of the domain and asymptotic solutions with localized initial data*, Russ. J. Math. Phys. **20** (2013), pp. 389–401.

[V L72] V. Lazutkin, *Existence of a continuum of closed invariant curves for a convex billiard*, Uspehi Mat. Nauk **27** (1972), pp. 201–202.

31.10 10:40-11:10

The Gaussian multiplicative chaos for the sine-process

Alexander I. Bufetov

Steklov Mathematical Institute of the RAS, Moscow

Grigori Olshanski [Ols11] posed the problem of deciding when two determinantal measures are mutually absolutely continuous and of finding the corresponding Radon—Nikodym derivative.

Olshanski proved that the determinantal point process on $\mathbb{Z}$ governed by the Gamma-kernel is quasi-invariant under the group of finite permutations of $\mathbb{Z}$ and computed the Radon—Nikodym derivative, a multiplicative functional given by a generalized Euler product.

In development of Olshanski's programme it has been shown ([Buf18], Sept. 2014) that determinantal point processes governed by integrable kernels satisfying a regularity condition are quasi-invariant under the group of finite permutations in the discrete case and under the group of diffeomorphisms with compact support in the continuous case.

The Radon—Nikodym derivative is found explicitly as a regularized multiplicative functional over pairs of particles of our configuration. The key point is the equivalence of reduced Palm measures of the same order for our processes (that are rigid in the sense of Ghosh and Peres [GP17]): in this case the Radon—Nikodym derivative is given by a regularized multiplicative functional over particles of our process, in other words, a random Euler product. In the particular case of the sine-process the square root of the Radon—Nikodym derivative coincides, up to a linear multiple, with the "stochastic zeta-function" studied by Chhaibi, Najnudel, Nikeghbali [CNN17].

Recall now that the Gaussian multplicative chaos is a random measure introduced by Mandelbrot, Peyrière and Kahane in development of the 1941 theory of the local structure of turbulence by Andrei Nikolaevich Kolmogorov [Kol41]. Since the work of Yan Fyodorov and his collaborators [FHK12, FK14, FKS16, FS16] the convergence to the Gaussian multiplicative chaos for characteristic polynomials of random matrices has been a subject of intense research (see, e.g., [Ber17, BWW, LN, Web15], and references therein).

The random Euler product obtained as the square root of the Radon—Nikodym derivative of Palm measures of a determinantal point process naturally arises in the problem of minimality of realizations of our process in the Hilbert space that governs it.

In joint work with Yanqi Qiu and Alexander Shamov [BQS21] it is proved that almost every realization of a determinantal point process governed by an orthogonal projection is a complete set for the underlying reproducing kernel Hilbert space. The result had been conjectured by Lyons and Peres; in the case of the discrete space, it had been proved by Lyons; in the rigid case and, in particular, for the sine process, by Ghosh [Gho15].

Our complete set is however not minimal: indeed, almost every realization of the sine process has excess 1 for the Paley—Wiener space, that is, the configuration stays complete and becomes minimal after one particle is removed [Buf]. The reason for the excess one is that the suitably rescaled random Euler product converges in distribution to the Gaussian multiplicative chaos.

[Ber17] N. Berestycki, *An elementary approach to Gaussian multiplicative chaos*, Electron. Commun. Probab. **22** (2017), Paper No. 27.

[BQS21] Alexander I. Bufetov, Yanqi Qiu, and Alexander Shamov, *Kernels of conditional determinantal measures and the Lyons–Peres completeness conjecture*, J. Eur. Math. Soc. **23**:5 (2021), pp. 1477–1519.

[Buf] Alexander I. Bufetov, *The sine-process has excess one*, arXiv:1912.13454, 57 pp.

[Buf18] Alexander I. Bufetov, *Quasi-symmetries of determinantal point processes*, Ann. Probab. **46**:2 (2018), pp. 956–1003.

[BWW] Nathanaël Berestycki, Christian Webb, and Mo Dick Wong, *Random Hermitian matrices and Gaussian multiplicative chaos*, arXiv:1701.03289, 61 pp.

[CNN17] Reda Chhaibi, Joseph Najnudel, and Ashkan Nikeghbali, *The Circular Unitary Ensemble and the Riemann zeta function: the microscopic landscape and a new approach to ratios*, Invent. math. **207** (2017), pp. 23–113.

[FHK12] Y. V. Fyodorov, G. A. Hiary, and J. P. Keating, *Freezing transition, characteristic polynomials of random matrices, and the Riemann zeta function*, Phys. Rev. Lett. **108** (2012), 170601, 5 pp.

[FK14] Y.V. Fyodorov and J. Keating, *Freezing transitions and extreme values: random matrix theory, and disordered landscapes*, Philos. Trans. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci. **372** (2014), no. 2007, 20120503.

[FKS16] Y.V. Fyodorov, B. A. Khoruzhenko, and N. Simm, *Fractional Brownian motion with Hurst index $H = 0$ and the Gaussian Unitary Ensemble*, Ann. Probab. **44**:4 (2016), pp. 2980–3031.

[FS16] Y.V. Fyodorov and N. Simm, *On the distribution of maximum value of the characteristic polynomial of GUE random matrices*, Nonlinearity **29** (2016), pp. 2837–2855.

[Gho15] Subhroshekhar Ghosh, *Determinantal processes and completeness of random exponentials: the critical case*, Probab. Th. Relat. Fields **163** (2015), pp. 643–665.

[GP17] Subhroshekhar Ghosh and Yuval Peres, *Rigidity and tolerance in point processes: Gaussian zeros and ginibre eigenvalues*, Duke Math. J. **166**:10 (2017), pp. 1789–1858.

[Kol41] A.N. Kolmogorov, *The local structure of turbulence in incompressible viscous fluid for very large Reynolds numbers*, Dokl. Akad. Nauk SSSR **30**:4 (1941).

[LN] Gaultier Lambert and Joseph Najnudel, *Subcritical multiplicative chaos and the characteristic polynomial of the $C\beta E$*, arXiv:2407.19817, 35 pp.

[Ols11] Grigori Olshanski, *The quasi-invariance property for the Gamma kernel determinantal measure*, Adv. Math. **226**:3 (2011), pp. 2305–2350.

[Web15] C. Webb, *The characteristic polynomial of a random unitary matrix and gaussian multiplicative chaos – the L^2-phase*, Electron. J. Probab. **20**:104 (2015).

Six problems in classical KAM Theory

28.10
13:00-13:30

Luigi Chierchia
University "Roma Tre" Rome, Italy

In a series of recent papers ([LL20], [LL23], [LL24], [LL]) we developed a "singular KAM Theory" which allows to investigate primary and secondary KAM tori exponentially close to the singularities (separatrices) arising near simple resonances in nearly–integrable generic Hamiltonian systems. In particular, such a theory allows to get upper bound on the measure of the "non–torus set", which, according to conjectures made by Arnold, Kozlov and Neishdadt in [AKN88] and [AKN06], are sharp up to a logarithmic correction, at least in the case of natural (or mechanical) Hamiltonian systems.
After briefly reviewing the main ideas and techniques on which Singular KAM Theory is based, I will discuss six open problems which naturally arise in this context.

Partially supported by the grant *NRR-M4C2-I1.1-PRIN 2022-PE1-Stability in Hamiltonian dynamics and beyond-F53D23002730006-Financed by E.U.-NextGenerationEU*

[AKN06] V. I. Arnol'd, V. V. Kozlov, and A. I. Neishtadt, *Mathematical aspects of classical and celestial mechanics*, third, vol. 3, Encyclopaedia of Mathematical Sciences, Berlin: Springer–Verlag, 2006.

[AKN88] V. I. Arnol'd, V. V. Kozlov, and A. I. Neishtadt, *Dynamical systems. III*, vol. 3, Encyclopaedia of Mathematical Sciences, Berlin: Springer–Verlag, 1988, pp. xiv+291.

[LL] L. Biasco and L. Chierchia, *Singular Kam Theory*, arXiv:2309.17041, 66 pp.

[LL20] L. Biasco and L. Chierchia, *On the topology of nearly–integrable hamiltonians at simple resonances*, Nonlinearity **33** (2020), pp. 3526–3567.

[LL23] L. Biasco and L. Chierchia, *Complex Arnol'd–Liouville maps*, Regular and Chaotic Dynamics **28**:4–5 (2023), pp. 395–424.

[LL24] L. Biasco and L. Chierchia, *Global properties of generic real-analytic nearly-integrable hamiltonian systems*, Journal of Differential Equations **385** (2024), pp. 325–361.

Undercompressive shocks for nonstrictly hyperbolic conservation laws

Anna Chugainova

Steklov Mathematical Institute of RAS, Moscow

Undercompressive shocks and their role in solving Riemann problem are studied. Solutions to a special system of two hyperbolic equations representing conservation laws are investigated. On the one hand, this system of equations makes it possible to demonstrate the non-standard solutions to the Riemann problem, on the other hand, this system of equations describes longitudinal-torsional waves in elastic rods. We use the traveling wave criterion for admissibility of shocks as the additional jump condition. If the dissipation parameters included in each of the equations of the system are different, then there are undercompressed waves.

We have numerically studied the asymptotics of the main types of solutions to the Riemann problem for the system of equations describing nonlinear longitudinal–torsional waves in viscoelastic media [Chu24]. The study has shown that the asymptotics of nonstationary solutions of the Riemann problem may contain undercompressive shock (nonclassical discontinuities). The solutions with a undercompressive shock are formed for a certain relation between the dissipation parameters appearing in the equations. If two dissipation parameters are identical (or their ratio is close to unity), then the asymptotics of the solution corresponds to the self-similar solution and does not contain undercompressive shocks. We have shown that, for different ratios of dissipation parameters, one can obtain different solutions to the Riemann problem for the same initial data.

[Chu24] A.P. Chugainova, *Riemann problem for longitudinal–torsional waves in nonlinear elastic rods*, Z. Angew. Math. Phys **75**:106 (2024).

Adiabatic approximation in quantum and classical mechanics, semi-rigid wall billiards and long nonlinear coastal waves

29.10
12:30-13:00

Sergey Dobrokhotov

Ishlinsky Institute for Problems in Mechanics of the RAS, Moscow

In the paper [DNT23], time-periodic solutions of a nonlinear system of shallow water equations in basins with shallow gentle shores localized in the vicinity of the coastline were constructed. In this work, the construction of such solutions is associated with special trajectories of a two-dimensional Hamiltonian system with a Hamiltonian $H = D(x_1, x_2)(p_1^2 + p_2^2)$, where the function D is the depth of the basin. We denote the coastline $\Gamma = \{D = 0\}$ and assume that $\nabla D|_\Gamma \neq 0$. As D turns to zero as $x \to \Gamma$ thus suitable solutions of the Hamiltonian system organize the so-called billiards with semi-rigid walls, woven from trajectories located between the standard caustics and the "non-standard" ones $Gamma$. (see also [BT24]). These billiards implies the asymptotic eigenfunctions of the linear operator $-\nabla(D(x)\nabla)$ for large eigenvalues and turn mentioned coastal long nonlinear waves. The existence of these billiards with semi-rigid walls is possible in the case of integrable Hamiltonian systems with Hamiltonian H, which practically does not happen in real situations. In this talk, we consider degenerate situations where "standard" caustics are very close to the coastline ("non-standard" caustics). Then "fast and slow" variables appear in the problem and we can apply the classical and quantum versions of the adiabatic approximation, the requirement of integrability disappears and it is always possible to construct effective asymptotic wave solutions having a small number of oscillations normal to the shore [DMV24] (which are analogs of Stokes and Ursell waves). The corresponding trajectories are strongly localized in the narrow vicinity of the coast, while they always enter the coastline and reflect from it at an angle of 90 degrees. Thus, we have asymptotic solutions similar to the "whispering gallery" type solutions known in acoustics, but at the same time for their existence due to a "degenerate" wall (coastline) the convexity of the two-dimensional region $(x : D(x) > 0)$ in which the pool is located is not required.

This is a joint work with Dmitrii Minenkov and Maria Votyakova.

The study was supported by the Russian Science Foundation grant No. 24-11-00213, https://rscf.ru/project/24-11-00213/

[BT24] S. Bolotin and D. Treschev, *Another billiard problem*, Russ. J. Math. Phys. **31**:1 (2024), pp. 50–59.

[DMV24] S. Dobrokhotov, D. Minenkov, and M. Votiakova, *Asymptotics of long nonlinear coastal waves in basins with gentle shores*, Russ. J. Math. Phys. **31**:1 (2024), pp. 79–93.

[DNT23] S. Dobrokhotov, V. Nazaikinskii, and A. Tsvetkova, *Nonlinear effects and run-up of coastal waves generated by billiards with semi-rigid walls in the framework of shallow water theory*, Proc. Steklov Inst. Math. **322** (2023), pp. 105–117.

Rigorous derivation of the wave kinetic equation in subcritical scaling without Feynman diagrams

31.10
13:10-13:40

Andrey Dymov

Steklov Mathematical Institute of RAS & HSE University & RUDN University, Moscow

Wave turbulence theory applies to describe a vast range of weakly nonlinear dispersive wave systems and has been intensively developing in physical works since 1960-th [ZLF92]. It claims that the total energy of the system is distributed over frequencies according to a nonlinear kinetic equation called the wave kinetic equation (which goes back to R. Peierls [Pei29]). During the last decade significant progress has been achieved in the problem of mathematical justification of the wave turbulence prediction. In particular, Yu. Deng and Z. Hani in [DH23] established it in the case when the dynamics of waves is given by a nonlinear Schrödinger equation. Due to our knowledge this is the only result establishing the wave turbulence prediction under the *critical* scaling (which is the "right one").

All existing results including the cited one rely on a straightforward decomposition of solutions to series, terms of which are computed iteratively and written via sums over a huge number of Feynman diagrams. This approach requires very delicate combinatorial analysis and leads to extremely technical and complicated proofs (length of [DH23] is about 180 pages). In [Dym] we establish the wave turbulence prediction for a stochastic perturbation of the NLS without evoking the Feynman diagram decomposition, relying instead on a robust inductive analysis of cumulants. This significantly reduces the technicality of the proof but for the moment we are able to proceed only in a *subcritical* scaling.

This is a development of joint works with Sergei Kuksin, Alberto Maiocchi and Sergei Vlăduţ [DK21, DK23, Dym+23].

This work is supported by the Ministry of Science and Higher Education of the Russian Federation (megagrant No. 075-15-2022-1115).

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Peierls' theory of thermal conductivity and the method of quasisolutions

31.10
13:40-14:00

Alexey Elokhin

HSE University, Moscow & Peoples' Friendship University of Russia (RUDN University)

In 1929 R. Peierls published his famous work [Pei29] devoted to the description of thermal conductivity in solids and derivation of the Fourier's law. He considered a lattice of anharmonic oscillators and showed at heuristic level of rigour that in the limit when the number of oscillators goes to infinity while the nonlinearity goes to zero, the distribution of energy over the Fourier modes (i.e. the energy spectrum) obeys a certain nonlinear kinetic equation. Despite significant efforts of mathematicians and mathematical physicists to rigorously derive this result, the problem remains unsolved.

Inspired by the Peierls' kinetic theory, during the second half of the XX-th century a parallel field, known as the wave turbulence theory, that focuses on studying weakly nonlinear wave systems, has been intensively developing. In the last decade in the problem of its rigorous justification was achieved significant progress.

In my talk I am aiming to discuss the results of the ongoing work with the setting similar to the Peierls's work, relying on the techniques that were recently developed in the works on the wave turbulence. Starting from the stochastically perturbed d-dimensional lattice of anharmonic oscillators, I will explain the derivation of the kinetic equation for energy spectrum of quasisolution, which is a certain approximation of the solution to the problem, and give a brief overview of the main steps and difficulties that we encountered in the process.

This is a joint work with Andrey Dymov and Alberto Maiocchi.

This research was supported by the Ministry of Science and Higher Education of the Russian Federation (megagrant No. 075-15-2022-1115).

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29.10
16:10-16:30

Dynamics of a spinning heavy disk in an ideal fluid

Mikhail Garbuz

Steklov Mathematical Institute of RAS, Moscow

The problem of motion of a homogeneous disk in the gravity field and in the infinite volume of an ideal incompressible fluid performing a vortex–free motion and resting at infinity is considered. The expression of the kinetic energy of the system has the form

$$2T = a_1u^2 + a_2v^2 + a_3w^2 + b_1p^2 + b_2q^2 + b_3r^2,$$

where u, v, w are the components of the velocity of O, and p, q, r are the components of the angular velocity of the disk. The constants a_i, b_i include the added masses and the added moments of inertia of the plate (see [Kir70]). The equations of motion for such a system was written in [Kir70], [Cha33].

Assume that at the initial moment the plate is horizontal, it has a vertical spin, and its center of mass has the horizontal velocity. Then, under the influence of gravity, the plate goes down and due to the lift force rotates around a lateral axis perpendicular to the directions of the initial velocity and initial spin. A similar effect was described in details in work [DK02]. At the same time, a gyroscopic effect appears. It motivates rotation of the plate around the velocity of the center of mass, and as a result a lateral motion of the plate.

We obtained the dynamics of the disk at small values of time (at the first moment after its "launch"). A numerical simulation of the motion of a thin spinning disk was performed. It confirmed the effect of lateral displacement.

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If a Minkowski Finsler billiard is projective, then it is a standard billiard

31.10
10:10-10:40

Alexey Glutsyuk

CNRS, UMR 5669 (UMPA, ENS de Lyon), France & HSE University, Moscow & Higher School of Modern Mathematics MIPT, Moscow

Projective and Finsler (Minkowski) billiards were introduced by S. Tabachnikov in [Tab97] and by E. Gutkin and S. Tabachnikov in [GT02]. These billiards generalize the standard billiard in a convex domain. In [Gluar] the speaker asked the question when a Finsler billiard is projective. He proved there that this happens if and only if up to affine transformation, the corresponding Minkowski norm is an euclidean norm, in which case the billiard is the standard billiard. The original proof given in [Gluar] was quite complicated and required C^6-smothness of boundaries of both the Finsler unit ball and the reflecting hypersurface and nontrivial asymptotic calculations. In the talk we present a simple direct proof valid in C^1-smooth case, obtained in our recent joint paper with Vladimir Matveev [GM].

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28.10
11:20-11:50

Motion of particles in the field of nonlinear wave packets and dark solitons in a fluid beneath an ice cover

Andrej Il'ichev

Steklov Mathemetical Institute of RAS, Moscow

A fluid layer of finite depth described by Euler's equations is considered. The ice cover is modeled by a geometrically non-linear elastic Kirchhoff-Love plate. The trajectories of liquid particles under the ice cover are found in the field of a nonlinear surface traveling waves of small, but finite amplitude, indicating the focusing and defocusing of nonlinear carier wave, namely, a solitary wave packet (a monochromatic wave under the envelope, the speed of which is equal to the speed of the envelope) and the so called dark soliton (the wave being a nonlinear product of a kink and periodic wave). The analysis uses explicit asymptotic expressions for solutions describing wave structures on the water-ice interface such as the solitary wave packet and dark soliton, as well as asymptotic solutions for the velocity field in the liquid column generated by these waves [ISS24].

This is a joint work with Alexander Savin.

The work of A.T. Il'ichev was supported by the Russian Science Foundation under grant no 19-71-30012, https://rscf.ru/en/project/ 23-71-33002/, and performed at Steklov Mathematical Institute of Russian Academy of Sciences.

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Construction of a three-dimensional bifurcation diagram in the problem of a rigid body rolling on a plane

31.10
12:40-13:10

Alexander Kilin

Ural Mathematical Center, Udmurt State University, Izhevsk

This paper investigates the problem of a heavy unbalanced sphere of radius R and mass m with axisymmetric mass distribution (a spherical top) rolling with partial slipping on a horizontal plane. It is assumed that there is no slipping of the sphere as it rolls in the direction of the projection of the symmetry axis onto the supporting plane. It is also assumed that, in the direction perpendicular to the above-mentioned one, the sphere can slip relative to the plane.

One way to fulfil this condition is to use a roller-bearing sphere (omnisphere). Such a sphere is a generalization of the omniwheel for which also only one nonholonomic constraint [BKM15] is imposed on the system. Such a sphere rolls along the meridian without slipping. In the direction perpendicular to the axis of the roller, the sphere can slide freely, which is made possible by rotation of the roller.

In [KI23] it is shown that the system under consideration admits a redundant set of first integrals and an invariant measure. This allows a reduction to a system with one degree of freedom, and all nonsingular trajectories are periodic functions of time.

The resulting system depends on the constants of four first integrals and two mass-geometric parameters. This greatly complicates the bifurcation analysis and the classification of different types of motion of the system which are presented in this paper.

This is a joint work with Tatiana Ivanova.

This work is supported by the framework of the state assignment or the Ministry of Science and Higher Education (No. FEWS-2020-0009).

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Nonlinear long standing waves with support bounded by caustics or localized in the vicinity of a two-link trajectory

29.10
13:00-13:20

Alexander Klevin

Ishlinsky Institute for Problems in Mechanics of RAS, Moscow

Long, nonbreaking waves in bounded water basins with a shore are described by the system (see, e.g., [Mei89])

$$\eta_t + \langle \nabla, (D(\mathbf{x}) + \eta)\mathbf{u} \rangle = 0, \quad \mathbf{u}_t + \langle \mathbf{u}, \nabla \rangle \mathbf{u} + g\nabla\eta = 0, \quad \mathbf{x} \in \mathbb{R}^2. \quad (1)$$

The free surface elevation $\eta(\mathbf{x}, t)$ and the averaged over the depth horizontal fluid velocity $\mathbf{u}(\mathbf{x}, t) = (\mathbf{u}_1(\mathbf{x}, t), \mathbf{u}_2(\mathbf{x}, t))$ are unknown functions, $D(\mathbf{x})$ is the function of basin depth. The solution is assumed to be defined in the time-dependent domain occupied by water $\{\eta(\mathbf{x}, t) + D(\mathbf{x}) \geq 0\}$. After the transformation defined by the parametric formulas (see [DMN22]) $\mathbf{x} = \mathbf{y} - N(\mathbf{y}, t)\rho(\mathbf{y})|\nabla_{\mathbf{y}} D(\mathbf{y})|^{-2}\nabla_{\mathbf{y}} D(\mathbf{y})$, $\eta = N(\mathbf{y}, t)$, $\mathbf{u} = \mathbf{U}(\mathbf{y}, t)$, where $\rho(y)$ is a cutoff function, the system (1) can be considered as the perturbation of the linear system

$$N_t + \langle \nabla_{\mathbf{y}}, D(\mathbf{y})\mathbf{U} \rangle = 0, \quad \mathbf{U}_t + g\nabla_{\mathbf{y}} N = 0.$$

The solution $(N(\mathbf{y}, t), \mathbf{U}(\mathbf{y}, t))$ is now assumed to be defined in the fixed domain $\{D(\mathbf{y}) \geq 0\}$. After eliminating of the $\mathbf{U}$-component and considering a time-harmonic solution of the form $N(\mathbf{y}, t) = e^{i\omega t}\psi(\mathbf{y})$ we obtain the eigenfunction problem $-\langle \nabla, gD(\mathbf{y})\nabla\psi \rangle = \omega^2\psi$. In the present paper ([KT23]) asymptotic (with respect to large ω) eigenfunctions $\psi(\mathbf{y})$ are constructed in the following two situations: 1) the domain is of a form of an elliptic annulus bounded by two confocal ellipses (a basin with an island); 2) there is a periodic trajectory with two reflections from the boundary of the domain.

This is a joint work with Anna Tsvetkova.

The study was supported by the Russian Science Foundation grant No. 24-11-00213, https://rscf.ru/project/24-11-00213/.

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28.10
10:00-10:40

Замечания о возвращаемости сумм Биркгофа

Валерий Васильевич Козлов

Математический институт имени В.А. Стеклова РАН

Рассматриваются сохраняющие меру, но не обязательно обратимые, эргодические преобразования компактного метрического пространства с мерой Каратеодори. Изучается поведение сумм Биркгофа для интегрируемых и п.в. ограниченных функций с нулевым средним значением по мере Каратеодори. Показано, что для почти всех точек метрического пространства существует бесконечная последовательность «моментов времени», вдоль которой суммы Биркгофа стремятся к нулю, и в те же моменты точки траектории сколь угодно близко подходят к своему начальному положению (как в теореме Пуанкаре о возвращении). В качестве примера рассмотрено преобразование $x \mapsto 2x \mod 1$ единичного отрезка $0 \leqslant x \leqslant 1$, тесно связанное с испытаниями Бернулли.

Long-time behaviour of trajectories for dynamical systems with random forces: non-Markov case

31.10
09:30-10:10

Sergei Kuksin

Université Paris Cité, Paris, France & Steklov Mathematical Institute of RAS, Moscow & RUDN University, Moscow

In my lecture I will present some results, recently obtained jointly with Armen Shirikyan. They deal with discrete time or continuous time dynamical systems with random force for unknown $\{u_k\}$ or $u(t)$ in some phase space H:

$$u_k = S(u_{k-1}, \eta_k^\omega), \quad k = 1, 2, \dots, \qquad u_0 = v; \tag{1}$$

or

$$\dot{u}(t) = F(u(t), \eta^\omega(t)) \quad t \geq 0, \qquad u(0) = v. \tag{2}$$

Here $\omega \in (\Omega, \mathcal{F}, \mathbf{P})$, and $\{\eta_k^\omega, k \in \mathbb{Z}\}$ and $\{\eta^\omega(t), t \geq 0\}$ are stationary in time bounded random processes, valued in some other space E; so $S : H \times E \to H$. The phase space H may be of infinite dimension. Thus equation (2) may be a nonlinear PDE, perturbed by a random force. Results for equations (1) and (2) are similar, and I will talk only about the discrete time case (1), where the statements are easier. Moreover, I will restrict myself to the case when spaces H and E are finite-dimensional:

$$H = \mathbb{R}^N, \quad E = \mathbb{R}^M, \quad M \geq N.$$

Then due to the boundedness, $\eta_k^\omega \in Y$ for all k, ω, for some compact subset Y of E. We assume that system (1) is dissipative in the sense that there exists a compact set $X \Subset H$ such that if $v \in X$, then $u_k \in X$ for all k and ω. Moreover, for any $v \in H$ there is $k(v)$ such that $u_k \in X$ for $k \geq k(v)$ and all ω. Our goal is to show that under certain restrictions system (1) is mixing. That is, in H there is a measure μ such that for any initial data v, $\mathcal{D}(u_k) \rightharpoonup \mu$ as $k \to \infty$ (here and below $\mathcal{D}$ signifies distribution and $\rightharpoonup$ stands for the weak convergence of measures).

The mixing in systems (1) and (2) was examined in a lot of works. In vast majority of them for discrete-time systems (1) it was assumed that the random variables $\eta_1, \eta_2, \dots$ are independent (and identically distributed). While for continuous-time systems (2) it was assumed that the random process $\eta(t)$ is a white noise, so (2) is a stochastic differential equation, or a stochastic PDE. The reason is that in this case solutions $\{u_k\}$ for systems (1) and solutions $u(t)$ for systems (2) are Markov processes in H, so the whole huge arsenal of Markov tools applies to study them. There are just a few works on mixing in systems (1) and (2) where processes η are not independent at different moments of time. There it is assumed that the processes are Gaussian, and the maps S and F which define these systems are very special. The goal of the research which I am presenting is to drop the independence assumption for a large class of systems (1), without assuming the Gaussianity. To state the result I impose some restrictions on the random force and on the equation. Concerning the random process η we assume that:

(RP 1) (mixing). The process $\{\eta_k\}$ is strongly exponentially mixing. (Roughly, it means that in some sense "correlations" of random variables η_k^ω and η_{k+t}^ω decays exponentially as $t \to \infty$.)

(RP 2) (Lipschitz regularity). For $l \in \mathbb{Z}$ denote by $\vec{\eta}_l$ the vector of the past of process $\{\eta_k\}$, $\vec{\eta}_l = (\dots, \eta_{l-1}, \eta_l) \in Y^\infty \subset E^\infty$. For a vector $\vec{\xi} \in Y^\infty$ consider the conditional distribution $Q(\vec{\xi})(\cdot)$ of random variable η_1^ω, given that the past $\vec{\eta}_0$ equals $\vec{\xi}$. This is a measure on the space $E = \mathbb{R}^M$, depending on $\vec{\xi}$. Assume that it has a density: $Q(\vec{\xi})(\cdot) = p_{\vec{\xi}}(x)dx$, where $\operatorname{supp} p_{\vec{\xi}}(\cdot) \subset Y$ and p is a Lipschitz function of $\vec{\xi}$ and x.

(RP 3) (recursiveness to the origin). For any given number $n \in \mathbb{N}$ and any given past $\vec{\xi} \in Y^\infty$, process $\{\eta_k, k \geq 1\}$ conditioned to the past $\vec{\eta}_0 = \vec{\xi}$, with a positive probability will sooner or later return to the small vicinity of the origin and will stay there time n.

Concerning the C^2-mapping S, apart from the dissipativity, we assume that:

(S 1) (linearised controllability). For any $u \in X \Subset H$ and $\eta \in Y \Subset E$

$$\text{linear mapping} \quad D_\eta S(u, \eta) : E \to H \quad \text{has dense image.}$$

(S 2) (dissipativity to the origin). $S(0, 0) = 0$, and if in (1) $\eta_k = 0$ for all k, then for any initial data $v \in H$ we have $u_k \to 0$ as $k \to \infty$.

Theorem 1. Suppose that hypothesis (RP 1)-(RP 3) and (S 1)-(S 2) hold. Then system (1) is exponentially mixing: in H exists a measure μ such that for any initial data $v \in H$, $\mathcal{D}u_k \rightharpoonup \mu$ exponentially fast.

The prove the result we first extend system (1) to a Markov process for vectors $U_k = (u_k, \vec{\eta}_k)$ and next prove the mixing for the extended system, using the Doeblin coupling and the method of Kantorovich functional, suggested in [KS12, Section 3.1.1]. The proof uses the

Newton method of quadratic convergence in a form, similar to that used by A.N.Kolmogorov to prove his celebrated theorem which originated the KAM theory. Cf. paper [KNS20] where a similar approach is used to handle related stochastic systems, also see [Kuk18, Section 5] for an informal presentation of the results in [KNS20].

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29.10
15:40-16:10

Application of the Kovacic algorithm to study the problem of the motion of a heavy gyrostat with a fixed point in the integrable Hess case

Alexander S. Kuleshov

Lomonosov Moscow State University

In 1890 W. Hess [W H90] found the new particular case of integrability of the Euler — Poisson equations of motion of a heavy rigid body with a fixed point. In 1963 L. N. Sretensky in his paper [L N63] proved that the particular case of integrability, similar to the Hess case, also exists in the problem of the motion of a heavy gyrostat — a heavy rigid body with a fixed point, which contains a rotating homogeneous flywheel. Further numerous generalizations of the classical Hess case were proposed [AI03]–[V A84], which take place during the motion of a heavy rigid body and a gyrostat with a fixed point in various force fields. The most general conditions of existence of the particular case of integrability, similar to the Hess case, in the problem of motion of a rigid body with a fixed point and a gyrostat in various force fields, were presented in the paper by A. A. Kosov [A A22].

The first studies that provided a qualitative description of the motion of a heavy rigid body in the integrable Hess case were published almost immediately after this case was found. In 1892 P. A. Nekrasov proved [P A92, P A96], that the solution of the problem of motion of a heavy rigid body with a fixed point in the integrable Hess case is reduced to the integration the second–order linear homogeneous differential equation with variable coefficients. A similar result regarding the problem of the motion of a heavy gyrostat in the integrable Hess case was presented by Sretensky [L N63].

In our talk we present the derivation of the corresponding second–order linear differential equation and reduce the coefficients of this equation to the to the form of rational functions. Then, using the Kovacic algorithm [J K86], we study the problem of the existence of liouvillian solutions of the corresponding second–order linear differential equation. We obtain the conditions for the parameters of the problem, under which the liouvillian solutions of the corresponding linear differential equation exist. Under these conditions equations of motion of a heavy gyrostat with a fixed point in the Hess case can be integrated in quadratures.

This work was supported financially by the Russian Science Foundation (project No RAI–RSF–2490).

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29.10
10:10-10:40

Spatial dynamics of some 6th-order ODE from the theory of phase transitions

Lev Lerman

HSE University

Bounded stationary (i.e. independent in time) solutions of a quasi-linear parabolic PDE

$$u_t = \frac{\partial^6 u}{\partial x^6} + A\frac{\partial^4 u}{\partial x^4} + B\frac{\partial^2 u}{\partial x^2} + u - u^3. \qquad (1)$$

are studied on the whole real line. This PDE (1) was proposed as a model of a formation and moving a phase front in media where the transitions from liquid to solid phase are possible [CF86]. Its stationary solutions represent a definite interest as a possibility to find spatial structures of various including complicated types. Earlier its stationary periodic solutions in some range of wave numbers were found by variational methods [TS02].

Stationary solutions of the equation (1) are described by a nonlinear ODE of the sixth order of the Euler-Lagrange-Poisson type and therefore are transformed to the Hamiltonian system with three degrees of freedom being in addition reversible with respect two linear involutions. The system has three symmetric equilibria, two of them $P_\pm$ are hyperbolic in some region of the parameter plane (A, B) (all eigenvalues have nonzero real parts). Here we investigate and discuss, combining methods of dynamical systems theory and numerical simulations, the orbit behavior near symmetric heteroclinic connections based on these equilibria and connecting them symmetric heteroclinic orbits relative to one or another involution.

It was found that both simple (periodic) and complicated orbit behavior are possible and this depends on the type of heteroclinic connections which are formed on these two basic equilibria and on the dimension of their leading subspaces. To this end, it was applied a theorem on the global center manifold in a neighborhood of the heterocilic connection generated by two saddle type symmetric equilibria $P_\pm$ and connecting them two heteroclinic orbits [Shi+98]. The heteroclinic orbits were found by numerical methods. When the leading subspaces for $P_\pm$ are one-dimensional, the global center manifold near the heteroclinic connection is two-dimensional symplectic and all nearby compact orbits are periodic ones accumulating to the connection. But if the leading subspaces of equilibria are two-dimensional, the global center manifold is four-dimensional symplectic and we deal in fact with the connection involving two saddle-foci with the related complicated dynamics on the four-dimensional invariant manifold. A characteristic feature of this situation is the existence of infinitely many homoclinic orbits of each saddle-focus-saddle.

For the third central symmetric equilibrium O at the origin we found the region in the parameter plane (A, B), when this equilibrium is of the saddle-focus-center type (its eigenvalues are a pair of pure imaginary $\pm i\omega$ and a quadruple of complex numbers $\pm\alpha \pm i\beta$, $\alpha\beta\omega \neq 0$). We have discovered the existence of homoclinic orbits of O and nearby families of periodic orbits. Due to some Turaev's theorem, a global center manifold cannot exist near a homoclinic orbits of this equilibrium and the reduction to a lesser dimension is not possible here. Nevertheless, a complicated behavior of nearby orbits exist here when some genericity condition holds for the linear Hamiltonian system linearized at the homoclinic solution of the saddle-focus-center [KL96]. Also some numerical findings show that near a homoclinic orbit of a saddle-focus-center in levels of the Hamiltonian close to the singular level containing O there are 2-elliptic periodic orbits (all four their multipliers belong to the unit circle in the complex plane $\mathbb{C}$) that says, if some inequalities hold, about restoring a regular (quasi-periodic) KAM behavior from the irregular complicated one.

This talk is based on results obtained in the joint recent paper with N.E. Kulagin [KL24].

The work was carried out in the framework of the Basic Research Programme of the HSE University.

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Hamiltonian theory for confinement of charged particles by magnetic fields

28.10
12:30-13:00

Robert MacKay

Mathematics Institute, University of Warwick, Coventry, UK

In a strong C^1 magnetic field B on a 3D Riemannian manifold M with metric g and compatible volume-form Ω, charged particle motion (charge e, mass m) has an adiabatic invariant, the magnetic moment $\mu = \frac{mv_\perp^2}{2|B|}$. The reduced dynamics for the guiding centre is a vector field X on the sub-bundle GM of T^*M spanned by the covector of the magnetic field. It has Hamiltonian formulation $i_X\omega = -dH$ with $H = \frac{1}{2}mv_\parallel^2 + \mu|B|$, $\omega = e\beta + d(mv_\parallel b^\flat)$, where $\beta = i_B\Omega$, $b = B/|B|$, and $b^\flat = i_b g$.

The ideal for confinement (at the level of single particles) is when guiding-centre motion has a Hamiltonian symmetry. The natural lift to GM of a vector field u on M is a symmetry of guiding-centre motion iff $L_u\beta = 0$, $L_u b^\flat = 0$ and $L_u|B| = 0$. Such u are called quasi-symmetries of the field.

In joint work with Burby and Kallinikos, we deduce strong restrictions on the pair (u, B) [BKM20, BKM]. Outstanding questions are whether every such u is a Killing field (i.e. $L_u g = 0$), and whether allowing velocity-dependent symmetries gives more possibilities.

This work was funded by a grant from the Simons Foundation (601970, RSM).

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31.10
12:10-12:40

Tensor invariants and conservation laws in mechanics

Ivan Mamaev

Kalashnikov Izhevsk State Technical University, Izhevsk

The influence of various conservation laws on the dynamics of the system is examined by considering different mechanical systems. In contrast to the standard conservation laws, which are given by first integrals of the system, the focus of my work is on conservation laws that are described by more complex tensor invariants (invariant measure, Poisson structures). Examples are given to show what dynamical phenomena are a consequence of the existence or absence of these conservation laws. In particular, some obstructions to Hamiltonization of dynamical systems are considered.

This is a joint work with Alexander Kilin and Ivan Bizyaev.

The work was carried out within the framework of the state assignment of the Ministry of Science and Higher Education of Russia (FZZN-2020-0011).

Birkhoff billiards in cones

29.10
15:00-15:40

Andrey E. Mironov
Sobolev Institute of Mathematics, SB RAS

Birkhoff billiards are dynamical systems that study the motion of a particle within a domain $D \subset \mathbb{R}^n$ with a piecewise smooth boundary ∂D. The particle moves freely at unit speed within D and reflects elastically off the boundary, with the angle of incidence equal to the angle of reflection. The theory of Birkhoff billiards has produced many remarkable results and unresolved problems, with the Birkhoff conjecture being one of the most intriguing.

In this report, we focus on Birkhoff billiards within a cone $K \subset \mathbb{R}^n$. We find that this billiard system always possesses a quadratic first integral in the velocity vector components:

Theorem 1. *Any cone in $\mathbb{R}^n$ admits the first billiard integral*

$$I = \sum_{i<j=1}^{n} m_{i,j}^2,$$

where $m_{i,j} := x_i v_j - x_j v_i$ for $i > j$ and $i, j = 1, \ldots, n$, and $v = (v_1, \ldots, v_n)$ is the velocity vector.

Using this result, we prove two key findings:

Theorem 2. *In a convex C^3-smooth cone, any billiard trajectory has only a finite number of reflections.*

Theorem 3. *There exist convex C^2-smooth cones in $\mathbb{R}^n$ where billiard trajectories can have infinitely many reflections in finite time.*

This is a joint work with Siyao Yin.

The work is supported by the Mathematical Center in Akademgorodok under the agreement No. 075-15-2022-281 with the Ministry of Science and Higher Education of the Russian Federation.

01.11
10:00-10:30

On probability of capture into 1:1 ground-track resonance while descending towards an asteroid

Anatoly Neishtadt

Loughborough University, UK & Space Research Institute, Russia

A spacecraft may be captured into a ground-track resonance while descending towards an asteroid using low-thrust propulsion. The probability of capture is typically estimated using Monte Carlo simulations. We have developed semi-analytic and analytic methodologies to estimate the probability of capture into a 1:1 ground-track resonance. The dynamics of the captured spacecraft and an escape from the capture are also described. As an example, estimations of the Dawn spacecraft's capture probability into a 1:1 ground-track resonance around Vesta are provided. The results are validated against numerical estimations based on Monte Carlo simulations.

This is a joint work with Wail Boumchita and Jinglang Feng.

Path optimization in inhomogeneous media by a direct approach

29.10
13:20-13:40

Igor A. Nosikov

P.G. Demidov Yaroslavl State University

A variant of the direct optimization method for point-to-point ray path is presented. The main features of this approach are stability in the case of a large divergence of the rays and locality - only rays connecting the start and end points are calculated. These circumstances make it possible to most effectively apply analytic-numerical formulas for the wave amplitude based on the Maslov operator. The main idea of the proposed variational approach is to successively optimize the ray path from some initial approximation to the desired optimum — the solution for which the objective function satisfies the stationarity principle. The presented method has the ability to determine both minima and saddle points of a given functional. A procedure for a systematic search for a set of stationary solutions based on the transitional properties of saddle points is implemented. In the framework of this work, the method is applied to calculate optimal paths in various fields [BT24, BK78], including tunneling effects and tsunami problem. Verification was carried out with the results of traditional ray tracing.

This is a joint work with Sergey Yu. Dobrokhotov and Anton A. Tolchennikov.

This work was supported by the Russian Science Foundation (project No. 21-71- 30011).

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29.10
09:30-10:10

Mathematical aspects of brain growth modeling

Pavel I. Plotnikov

Lavryentyev Institute of Hydrodynamics, Novosibirsk

The work is devoted to the analysis of the mathematical model of the volumetric growth of incompressible neo-Hookean material. Models of this kind are used in order to describe the evolution of the human brain under the action of an external load. The model consists of a nonlinear elastostatic equation and an evolution equation for a deformation vector field $\mathbf{u}$, pressure function p, and growth factor w. We study the structure of the governing equations and show that they formally can be reduced to a gradient flow of the marginal energy for the growth factor. We study in many details a variety of the homeostatic states. In particular, we show that the space of homeostatic deformation fields $\mathbf{u}$ coincides with the Möbius group of conformal transforms in $\mathbb{R}^3$. Moreover, the corresponding homeostatic growth factors equal conformal factors e^f of the homeostatic deformation fields. Note that the homeostatic conformal factors forms 4-dimensional manifold in the space of smooth functions. We modify the Stopelli-Valent method in order to prove the local well-posedness of the initial-boundary value traction problem in a neighborhood of the homeostatic manifold. The main conclusion is the multiplicity of gradient flows for the growth factor.

In the work, we also study the long time behavior of solutions to the mathematical model of the volumetric growth of incompressible neo-Hookean material. We take an interest in the following question, which is of practical significance. Assume that a material is under the the temporary load vanishing after some moment (hydrocephalus natural or induced). This means that the external forces in the elasticity equations vanish for all sufficiently large values of the temporal variable. The question arises as to whether the growth process is reversible. In other words, does the mechanical system return to the initial configuration. More general question is the converges of the growth process to some final homeostatic state. We prove that for every infinitely smooth solutions $\mathbf{u}(t)$, $p(t)$, $w(t)$ to the governing equations, there is a conformal mapping $\mathbf{u}_\infty$ with the conformal factor e^f such that the orbit $w(t) \to e^f$ as $t \to \infty$ in every Sobolev space. The proof is based on the extension the Łojasiewicz-Simon theory to the case of gradient flows with multiple potentials, which Hessians are not Fredholm operators of index zero.

NMS-flows on 4-manifolds

28.10
15:00-15:30

Olga Pochinka
HSE University, Nizhny Novgorod

In this talk, we consider the so-called *NMS-flows* f^t, that is, *non-singular* (without fixed points) Morse-Smale flows given on closed orientable n-manifolds M^n, $n \geq 2$. The non-wandering set of such a flow consists of a finite number of periodic hyperbolic orbits.

In the case of a small number of orbits, the known invariants can be significantly simplified and, most importantly, the classification task can be brought to realization by describing the admissibility of the obtained invariants. In [PS22], an exhaustive classification of flows with two orbits on arbitrary closed n-manifolds was obtained. In the work [Uma90], the problem of classification for three-dimensional Morse-Smale flows with a finite number of special trajectories is solved. The topological equivalence of non-singular flows, under assumptions of different generality, on the 3-sphere, is obtained, for example, in [Fra85], [Yu16].

In [GPS23], it was found that the only orientable 4-manifold admitting NMS-flows with exactly one saddle periodic orbit, assuming that it is *twisted* (its invariant manifolds are non-orientable), is $\mathbb{S}^3 \times \mathbb{S}^1$. It is also proved there that such flows are divided into eight equivalence classes. We should immediately note that in the case of a non-twisted orbit, the number of equivalence classes of such flows is infinite, as follows from the work [PS20], and among them there are flows with wildly embedded invariant manifolds of the saddle orbit.

This paper is devoted to the topological equivalence of four-dimensional NMS-flows with exactly one saddle periodic orbit, assuming that it is untwisted.

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On some geometric ideas in the method of averaging

29.10
12:00-12:30

Ivan Yu. Polekhin

Steklov Mathematical Institute of RAS, Moscow & Moscow Institute of Physics and Technology & Lomonosov Moscow State University & P.G. Demidov Yaroslavl State University

We present a topological-analytical method for proving some results of the N.N. Bogolyubov method of averaging [Bog45, BM63] for the case in which the time interval is infinite. The essence of the method is to combine topological methods of proving the existence of a periodic solution or a solution which never leaves some subset of the phase space, with the theorem of N.N. Bogolyubov on the averaging on a finite time interval. In particular, the considered approach allows us to abandon the non-degeneracy condition on the Jacobi matrix from the classical theorems of the averaging method.

We will consider the averaging procedure in the periodic and non-periodic cases; we will discuss and explain the geometry that underlies the classical results on averaging in these two cases. We will also briefly discuss the method of averaging for a part of variables and some results concerning the averaging in a neighborhood of an elliptic equilibrium in the non-periodic case. In particular, we will introduce a procedure of hyperbolization of an elliptic equilibrium, with the help of which new results on averaging over an infinite time interval can be obtained.

This work was performed at the Steklov International Mathematical Center and supported by the Ministry of Science and Higher Education of the Russian Federation (agreement no. 075-15-2022-265).

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01.11
10:30-11:00

Admissibility of discontinuities in the solutions of a hyperbolic 2×2 system of conservation laws

Ruzana Polekhina

Steklov Mathematical Institute of RAS, Moscow & Keldysh Institute of Applied Mathematics RAS, Moscow

The work is devoted to the study of the problem of admissibility of discontinuities in the solutions of a hyperbolic system of two conservation laws describing quasitransverse waves in nonlinearly elastic weakly anisotropic media obtained in [Kul86]. The standard viscous regularization method is applied to the defining system of equations. Regularization leads to the situation where two different viscosity profiles may correspond to the discontinuity [KS95, CP23].

The solutions of the system of equations depend on the sign of the parameter multiplying the fourth-order nonlinear term of the flux function. In this work the study of the stability of viscosity profiles has been carried out for positive and negative values of the nonlinearity parameter $\varkappa$. A single stable structure is found for each of these cases. If $\varkappa > 0$, then the "upper" structure is stable, while, if $\varkappa < 0$, then the "lower" structure is stable. The analysis of the linear (spectral) stability of these two profiles has shown that one of them is stable while the other is unstable.

We have numerically solved the Riemann problem in the case when the initial discontinuity corresponds to two different viscosity profiles. The results of calculations have shown that the asymptotics of a non-stationary solution of the Riemann problem represents a linearly stable viscosity profile. A linearly unstable viscosity profile is not a solution to the Riemann problem.

This conclusion demonstrates that the definition of admissibility of a discontinuity should include the requirement of stability of the viscosity profile.

This is a joint work with Anna Chugainova.

This work was supported by the Russian Science Foundation under grant no. 19-71-30012, https://rscf.ru/en/project/19-71-30012/

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29.10
10:40-11:10

Existence of localized motions near an unstable equilibrium position

Tatiana Salnikova

Lomonosov Moscow State University

We discuss a new type of motion—motion localized in the vicinity of an unstable equilibrium position. Let us consider a dynamical system whose equilibrium position is non-degenerate and unstable in Lyapunov sense, and its degree of instability is greater than zero and less than the number of degrees of freedom. The energy at the equilibrium position is assumed to be zero. It is shown that for any sufficiently small positive value of the total energy of the system, there is a motion of the system with a given energy value that begins at the boundary of the region where motion is possible and does not leave a small neighborhood of the equilibrium position. We call such motions as localized motions.

An essential condition for the presence of such movements is the limitation of system movements in “unstable directions.” For natural systems with gyroscopic and dissipative forces, this is ensured by the conservation or non-increase of the total mechanical energy. The use of topological methods [Bor67] in the analysis of such motions makes possible to abandon the condition of analyticity of the first integrals and the condition of non-resonance of purely imaginary roots of the characteristic equation. The presence of time-dependent gyroscopic and dissipative forces, as well as forces with incomplete dissipation, does not interfere with the proof of the existence localized solutions.

As an example, we consider the restricted circular three-body problem. Two massive bodies, due to mutual gravitational attraction, move in the same plane in circular orbits with a constant speed around their center of mass. A third body of sufficiently small mass does not affect the motion of massive bodies, it is under the influence of attractive forces to two massive bodies On a rotating orbital plane with a beginning at this center of mass and one of the axes passing through the centers of massive bodies, taking into account inertial forces, there are five positions of relative equilibrium of a small body - libration points. Three collinear libration points have a degree of instability equal to unity, therefore, according to the Kelvin-Chetaev theorem, they cannot be stabilized by adding dissipative and gyroscopic forces. Nevertheless, in accordance with the above, localized trajectories should exist near these unstable collinear libration points. Numerical simulations for the parameters of the Earth-Moon system convincingly illustrate our theoretical study.

This is a joint work with Eugene Kugushev.

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29.10
16:30-16:50

Morse-Smale systems on simply connected manifolds

Ilya Saraev

HSE University, Nizhny Novgorod

A Morse-Smale system dynamical system given on a closed manifold M^n is a structurally stable system whose non-wandering set consists of finite number of orbits. In 1961, Smale obtained inequalities connecting the number of fixed point and periodic orbits of such systems and Betty numbers of the ambient manifold similar to Morse inequalities for Morse function. In particular, it was proven, that if a Morse-Smale diffeomorphism $f : M^n \to M^n$ does not have periodic points with one-dimensional unstable invariant manifolds, then the homology group $H_1(M^n)$ is trivial. However, for $n \geq 3$ it does not mean that the fundamental group $\pi_1(M^n)$ of M^n is trivial. The following theorem enhances this result.

Theorem 1. *Let M^n be a closed manifold, $n \geq 3$, and $f : M^n \to M^n$ is a Morse-Smale diffeomorphisms such that $(n-1)$-dimensional invariant manifold of arbitrary saddle periodic point does not contain heteroclinic submanifolds. If $k_1 = 0$ then M^n is simply connected.*

For $n = 3$, the unique simply connected manifold is the sphere S^3 and Theorem 1 follows from [Bon+02] and [OE]. For $n \geq 4$, there are a numerous of simply connected manifolds not homeomorphic to sphere (for instance, $S^k \times S^l$, $k, l \geq 2, k + l = n$), but a complete classification of smooth simply connected manifolds is known only for $n = 4$ due to non-trivial results of Rochlin, Freedman, Donaldson and Furuta (see [A05] for references).

To proof Theorem 1, we obtain the following topological version of well known smooth result that can be of independent interest:

Proposition 1. *Let Q^{n-1}, M^n be closed topological manifolds, Q^{n-1} is simply connected and locally flat in M^n. Then there exists an embedding $e : Q^{n-1} \times [-1, 1] \to M^n$ such that $e(Q^{n-1} \times \{0\}) = Q^{n-1}$.*

This work was prepared within the framework of the project “International academic cooperation” HSE University.

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01.11
11:00-11:40

Semi-classical asymptotics for Dirac equations in abruptly varying external fields

Andrei Shafarevich

Lomonosov Moscow State University

We study structure of Lagrangian surfaces, corresponding to semi-classical solutions of Dirac equations. We assume that external electromagnetic field varies abruptly near certain hypersurface in Minkowsky space-time. The main result is asymptotic series for the solution of the Cauchy problem. We prove that the corresponding Lagrangian surface is divided into several parts, corresponding to reflected and transmitted waves. We also discuss the famous Klein effect in non-stationary semi-classical setting. The talk is based on the joint work with Anna Allilueva.

Quasi-satellites and mini-moons

28.10
16:20-16:50

Vladislav Sidorenko

Keldysh Institute of Applied Mathematics RAS, Moscow & Moscow Institute of Physics and Technology

In recent decades, there has been a rapid development of means and methods of astronomical observations. This has led to more than just an increase in the number of discovered asteroids. Previously unknown types of their dynamic behavior were revealed. As one of the most unexpected events, we can consider the discovery of asteroids that combine motion around the Sun with a long stay in the vicinity of one of the planets. Depending on whether the asteroid crosses the Hill sphere of the planet or not, such objects are divided into two classes: quasi-satellites [Kog90] and mini-moons [BJa14].

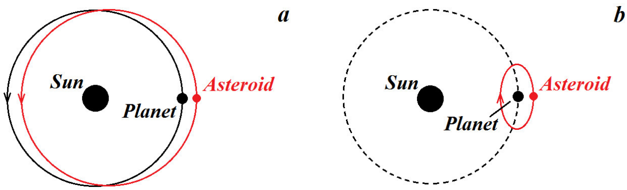

Figure 1: The orbital motion of a quasi-satellite and its host planet. Panel **a**: a Sun-centered reference frame that preserves the orientation in the absolute space. The quasi-satellite and the planet move around the Sun with the same orbital period in elliptic and in circular orbits respectively. Panel **b**: a Sun-centered frame rotating with the mean orbital motion of the planet

A quasi-satellite is an object that moves in the vicinity of a planet at a distance significantly less than the distance from this planet to the host star, and at the same time always remains outside its Hill sphere. The quasi-satellite mode of orbital motion is realized at 1:1 resonance of the mean motions of this object and the planet and, under certain conditions, it can transform into other modes of resonant motions (typically, into a horseshoe mode). In the quasi-satellite mode the asteroid's motion is weakly perturbed heliocentric (Fig. 1). This makes it possible to apply perturbation theory for analytical studies of quasi-satellite motions [MI97, Nam99, NCM99, Sid+14].

To date, eight asteroids are known to be quasi-satellites of the Earth: (164207) 2004GU$_9$, (277810) 2006FV$_{35}$, 2013LX$_{28}$, 2014OL$_{339}$, (469219) Kamoóalewa, 2020PP$_1$, 2022YG, 2023FW$_{13}$. We discuss the qualitative properties of the dynamics of these quasi-satellites using simple models: the restricted circular three-body problem "Sun-Earth-asteroid", averaged taking into account 1:1 mean motions resonance, and its modification, where the influence of other planets on the motion of the asteroid is added.

The objects, called mini-moons, orbit the planet several times in orbits that substantially intersect the Hill sphere. It is known that the Earth's mini-moons were asteroids 2006RH$_{120}$ and 2020CD$_3$. The motion of mini-moons is perturbed significantly more than the motion of quasi-satellites. Strong disturbances make it difficult to study the dynamics of mini-moons using analytical methods. A number of important results about the possible number of mini-moons near the Earth and the duration of stay of objects in the mini-moon regime were obtained using numerical calculations.

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Central extensions of Lie algebras, dynamical systems, and symplectic nilmanifolds

28.10
10:40-11:20

Iskander Taimanov

Sobolev Institute of Mathematics, Novosibirsk

The connections between Euler's equations on central extensions of Lie algebras and Euler's equations on the original, extended algebras are described. A special infinite sequence of central extensions of nilpotent Lie algebras constructed from the Lie algebra of formal vector fields on the line is considered, and the orbits of coadjoint representations for these algebras are described. By using the compact nilmanifolds constructed from these algebras by I.K. Babenko and the author, it is shown that covering Lie groups for symplectic nilmanifolds can have any rank as solvable Lie groups.

01.11
12:10-

On quantum Floquet theorem

Dmitry Treschev

Steklov Mathematical Institute of RAS, Moscow

I study a quantum particle on a circle in the force field of a periodic in time potential. The corresponding Schrödinger equation is a linear differential equation on the Hilbert space H of square integrable functions on the circle. I prove that the monodromy operator for this system is a sum of a diagonal operator and a compact one.

Lagrangian manifolds with degenerate fold and applications to the theory of wave beams

29.10
13:40-14:00

Anna Tsvetkova

Ishlinsky Institute for Problems in Mechanics RAS, Moscow

We discuss a geometric approach based on the Maslov canonical operator theory [MF01] to constructing the asymptotics of solutions of (pseudo)differential problems. The problem is associated with a geometric object – a Lagrangian manifold in the phase space. The type of singularity on this manifold determines the type of special function in terms of which the asymptotics can be expressed. In particular, singularities of the type of degenerate fold correspond to an asymptotic expression in terms of the Bessel function J_0.

This kind of singularities arises in the theory of wave beams, in particular, for Bessel and Laguerre-Gauss beams. Laguerre-Gauss beams are solutions of the three-dimensional Helmholtz equation in the paraxial approximation (which can be considered as the Shrödinger equation). The considered beams are the product of the Gaussian exponent and the Laguerre polynomials. The discussed approach to constructing asymptotics is based on studying the dynamics of the initial Lagrangian manifold along a Hamiltonian vector field with a Hamiltonian corresponding to the Schrödinger equation. It allows us to obtain an effective asymptotics of such beams in terms of Airy and Bessel functions of compound argument [DNT23].

One of the advantages of the discussed approach is that it is quiet universal. In particular, it allows to abandon the paraxial approximation and consider the original Helmholtz equation. In the talk the global asymptotics in terms of special functions of the solution of the Helmholtz equation with "initial" conditions generated by Laguerre-Gauss beams will be also given.

The work was supported by Russian Science Foundation (project 24-11-00213) https://rscf.ru/project/24-11-00213/

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31.10
11:10-11:40

Codimension one basic sets of Axiom A flows

Evgeny V. Zhuzhoma

National Research University "Higher School of Economics" Nizhny Novgorod

Axiom A systems (in short, A-systems) were introduced in Dynamical Systems by Steve Smale as the systems whose non-wandering set is hyperbolic and the closure of periodic points. Well known that a non-wandering set of such system splits into so-called basic sets (invariant and transitive components). Basic sets of maximal dimension are codimension one ones.

We consider A-flows (dynamical systems with continuous time) with non-mixing codimension one basic sets on closed manifolds. One shows that such basic set is either an attractor or repeller. We describe a topological structure of special compactification of basin of attractors. Especially, we consider two-dimensional attractors on 3-manifolds.

This is a joint work with Vladislav Medvedev.

This work was prepared within the framework of the project "International academic cooperation" Higher School of Economics (HSE University).

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