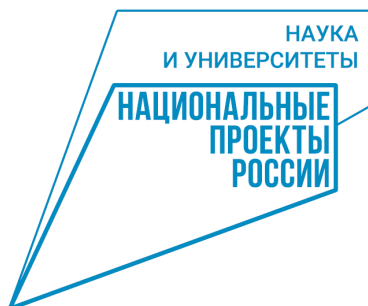


# 2024 Sino-Russian Bilateral International Mathematics Conference

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Shanghai University, Shanghai, P.R. China



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## Abstracts of plenary talks

**Speaker:** Alexander Aptekarev (Keldysh Institute of Applied Mathematics Russian Academy of Sciences)

**Title :** Laplacian growth, normal random matrices and multiple orthogonal polynomials

**Abstract:** We discuss a connection between ensembles of random matrices and asymptotics of the orthogonal polynomials. Our main attention will be devoted to the normal matrices ensembles, which have many interesting applications (Laplacian growth, Diffusion limited aggregation) An important feature of the orthogonal polynomials ensembles of random matrices is that the joint probability density of their eigenvalues is represented by means of the determinants composed by Christoffel-Darboux (CD) kernels of orthogonal polynomials or their generalizations. For the normal matrices ensembles the corresponded CD kernel is taken for polynomials orthogonal with respect to an area measure. We show that for some special cases of the normal random matrices (related with discrete Painlevé equation) these polynomials are the multiple orthogonal polynomials. This fact makes their asymptotical analysis much easier.

**Speaker:** Roman Bessonov (St.-Petersburg State University)

**Title:** Discrete NLSE and Schur's algorithm for analytic functions

**Abstract:** The discrete nonlinear Schrodinger equation is one of the simplest integrable models of mathematical physics. Having in mind applications in nonlinear optics, we consider the problem of its fast numerical solution with given accuracy. We show how Schur's algorithm for analytic functions can be used to overcome the main existing difficulties: (1) non-injectivity of the inverse scattering transform (IST) for general initial data; (2) lack of convergence estimates for IST-based algorithms for compactly supported initial data. Joint work with Pavel Gubkin (St.Petersburg).

**Speaker:** Andrei Bogatyrev (Marchuk Institute of Computational Mathematics of Russian Academy of Sciences, Lomonosov Moscow State University, Moscow Center for Fundamental and Applied Mathematics)

**Title:** Chebyshev Ansatz for multiband filtering: a review

**Abstract:** We consider an application of Riemann surfaces and their moduli spaces to a problem of electrical engineering. The synthesis of optimal filters brings us to a certain uniform rational approximation problem which is a multiband extension of celebrated 3rd and 4th Zolotarev's problems. Many prominent mathematicians including E.I.Zolotarev, N.I.Akhiezer, E.Stiefel and A.A.Gonchar contributed to this topic. With the use of a certain algebro-geometric formula (Ansatz) the rational optimization problem is reduced to the solution of a relatively small set of transcendental equations on moduli of Riemann surfaces. The complexity of this approach does not depend on the degree of the filter, but rather on the number of its stop- and pass-bands.

**Speaker:** Yifei Chen (Academy of Mathematics and Systems Sciences, Chinese Academy of Sciences)

**Title:** Jordan property of automorphism groups of surfaces of positive characteristic

**Abstract:** A classical theorem of C. Jordan asserts the general linear group  $GL_n$  over a field of characteristic zero is Jordan. That is, any finite subgroup of  $G$  contains a normal abelian subgroup of index at most  $J$ , where  $J$  is an integer only depends on the group  $G$ . For the Cremona group of rank  $n$ , which is the birational automorphism group of the projective space of dim  $n$ , J.-P. Serre proved that the Cremona group of rank 2 has Jordan property. Serre conjectured that the Cremona group of rank  $n$  has Jordan property. Prokhorov and Shramov proved the Cremona group of rank 3 has Jordan property, and they pointed out Serre's conjecture holds if the boundedness of Fano varieties conjecture (BAB conjecture) holds. As the BAB conjecture is proved by the Fields medalist Caucher Birkar, Serre's conjecture holds. In this talk, we will discuss Jordan property for automorphism groups of surfaces of positive characteristic. This is a joint work with C. Shramov.

**Speaker:** Andrei Domrin (Lomonosov Moscow State University)

**Title:** Compatible equations and meromorphic extension

**Abstract:** The set of partial differential equations of the form  $u_t = u_n + P(u, u_1, \dots, u_{n-1})$ , where  $n$  is a positive integer,  $u(x, t)$  is the unknown function,  $u_t$  is the derivative with respect to  $t$ ,  $u_j$  is the  $j$ th derivative with respect to  $x$ , and  $x$  is a polynomial without constant and linear terms, splits into equivalence classes with respect to formal compatibility. Up to rescaling  $x, t, u$ , there are exactly ten non-singleton equivalence classes of positively weighted homogeneous equations. We prove that the meromorphic extension property (every local holomorphic solution  $u(x, t)$  extends to a globally meromorphic function of  $x$  for every fixed  $t$ ) holds for every equation in nine of these classes and none in the tenth.

**Speaker:** Jixiang Fu (Fudan University)

**Title:** The LYZ equation

**Abstract:** The LYZ equation is called the deformed Hermitian Yang-Mills equation in literature. In this talk we first recall the recent progress of the LYZ equation, and then introduce a new flow solving the LYZ equation in Kahler geometry. This is a joint work with S.-T. Yau and Dekai Zhang.

**Speaker:** Nikolay Kruzhilin (Steklov Mathematical Institute of Russian Academy of Sciences)

**Title:** Fundamental theorem of projective geometry and its complex-geometric applications

**Abstract:** Local holomorphic versions of the fundamental theorem of projective geometry are considered. Some of its applications to the CR-geometry of real surfaces in complex spaces are discussed, namely, the equivalence problem for tubular real hypersurfaces and CR-maps of Hermitian quadrics.

**Speaker:** Jinsong Liu (Academy of Mathematics and Systems Science, Chinese Academy of Sciences)

**Title:** Riesz conjugate function theorem for harmonic quasiconformal mappings

**Abstract:** In this talk we will introduce the Riesz conjugate functions theorem for planar harmonic  $K$ -quasiregular mappings (when  $1 < p \leq 2$ ) and harmonic  $K$ -quasiconformal mappings (when  $2 < p < \infty$ ) in the unit disk. Moreover, if  $K = 1$ , then our constant coincides with the classical analytic case. For the  $n$ -dimensional case ( $n > 2$ ), we also obtain the Riesz conjugate functions theorem for invariant harmonic  $K$ -quasiregular mappings when  $1 < p \leq 2$ . This is a joint work with Dr. Jianfeng Zhu.

**Speaker:** Ngaiming Mok (The University of Hong Kong)

**Title:** From holomorphic isometries to functional transcendence results for quotients of bounded symmetric domains by cocompact lattices

**Abstract:** Consider a Kähler manifold  $(X, g)$ . When  $g$  can be expanded in power series, in his seminal work on holomorphic isometries Eugenio Calabi introduced the notion of the *diastasis* and proved powerful extension theorems on holomorphic isometries from Kähler manifolds into space forms such as the projective space equipped with the Fubini-Study metric. On a bounded domain  $U \Subset \mathbb{C}^n$  we denote by  $ds_U^2$  the Bergman metric on  $U$  (which is Kähler). Among bounded domains there are the bounded symmetric domains  $\Omega$  classified by Élie Cartan such that  $(\Omega, ds_\Omega^2)$  are symmetric in the sense of Riemannian geometry,  $\Omega = G/K$  in standard notation. Here  $U \Subset \mathbb{C}^n$  in their standard realizations are semi-algebraic, i.e., defined by algebraic inequalities in the  $2n$  real Euclidean coordinates underlying  $\mathbb{C}^n \cong \mathbb{R}^{2n}$ . By an irreducible algebraic subvariety of  $\Omega$  we mean an irreducible component of the intersection  $V \cap \Omega$  of an affine algebraic subvariety  $V \subset \mathbb{C}^n$  with the bounded symmetric domain  $\Omega \Subset \mathbb{C}^n$ . In this lecture I will explain: (1) *how the study of holomorphic isometries between bounded domains was motivated by problems in arithmetic dynamics*, (2) *how their solutions were generalized to yield algebraicity results for holomorphic isometries with respect to the Bergman metric*, (3)

*how the study of the asymptotic behavior of holomorphic isometries of the Poincaré disk led to a uniformization theorem for projective varieties covered by algebraic subvarieties of  $\Omega$ , and (4) how the latter serves as a starting point for research in functional transcendence theory concerning  $X_\Gamma = \Omega/\Gamma$ , where  $\Gamma \subset G$  is an arbitrary lattice.*

In the special case of arithmetic lattices, (4) has been settled yielding the *Ax-Schanuel theorem on Shimura varieties* (with extensive generalizations by now) by Mok-Pila-Tsimerman (2019), using techniques involving in particular model theory from mathematical logic, techniques which are no longer available for arbitrary lattices. In the case where  $X_\Gamma = \Omega/\Gamma$  and  $\Gamma \subset G$  is an arbitrary cocompact lattice, I will explain how functional transcendence results concerning  $X$  can be proven using analytic techniques starting with the rescaling method on subvarieties of a bounded symmetric domain exiting  $\partial\Omega$ .

**Speaker:** Tuen Wai Ng (The University of Hong Kong)

**Title:** Fridman Function, Injectivity Radius Function and Squeezing Function

**Abstract:** The In this talk, we will introduce Fridman function, injectivity radius function and squeezing function for a bounded domain in the n-dimensional complex Euclidean space. We then explain how these biholomorphic invariants are related to each other. Finally, we give the explicit expression of these functions for an annulus in the complex plane. This is a joint work with Jonathan Tsai and Chiu Chak Tang.

**Speaker:** Andrei Okounkov (Columbia University)

**Title:**

**Abstract:**

**Speaker:** Denis Osipov (Steklov Mathematical Institute of Russian Academy of Sciences, National Research University Higher School of Economics, National University of Science and Technology MISIS)

**Title:** Bott cocycle and Riemann-Roch theorem

**Abstract:** The Bott cocycle (or the Bott-Thurston cocycle) is a 2-cocycle on the group of orientation preserving diffeomorphisms of the circle, and the Lie algebra version of this cocycle is the Gelfand-Fuks cocycle that defines the Virasoro algebra. I will speak about the formal version of the Bott cocycle. This is the 2-cocycle on the group of continuous  $A$ -automorphisms of the algebra  $A((t))$  of Laurent series over an arbitrary commutative ring  $A$  with values in the group of invertible elements of  $A$ . Moreover, I will consider the group that is the semidirect product of this group (the group of automorphisms) with the group of invertible elements of  $A((t))$ . Besides, I will write other explicit 2-cocycles on this group in the spirit of the formal Bott cocycle. I will calculate and relate some linear combination of these 2-cocycles in the second cohomology group with another 2-cocycle that is defined by means of determinants of infinite matrices or relative determinants of  $A$ -submodules in  $A((t))$  when  $A$  contains the field of rational numbers. The equality obtained in the second cohomology group is the formal version of the Deligne-Riemann-Roch theorem for a line bundle on a family of projective curves.

**Speaker:** Vladimir Peller (St.-Petersburg Department of Steklov Mathematical Institute of Russian Academy of Sciences)

**Title:** The behaviour of functions of operators under relatively bounded and relatively trace class perturbations

**Abstract:** A self-adjoint operator  $K$  is said to be a relatively bounded perturbation of a self-adjoint operator  $A$  if  $\|Kx\| \leq \text{const}(\|x\| + \|Ax\|)$  for an arbitrary vector  $x$  in the domain of  $A$ . In a similar way we can define relatively trace class perturbations. I am going to consider the problem of the behaviour of functions of operators under such perturbations. Sharp estimates for the norms of  $f(A+K) - f(A)$  will be obtained in the case of relatively bounded perturbations  $K$ . Also, we obtain a trace formula in the case of relatively trace class perturbations. The research is supported by Russian Science Foundation, grant number 23-11-00153.



**Speaker:** Vladimir Popov (Steklov Mathematical Institute of Russian Academy of Sciences)

**Title:** Around the cancellation problems in algebraic geometry

**Abstract:** The purpose of the talk is to discuss several topics inspired by the following question posed in the early eighties of the last century and still remaining open: if the product of an algebraic variety and an affine space is isomorphic to an affine space, then is it true that this variety itself is isomorphic to an affine space?

**Speaker:** Yury Prokhorov (Steklov Mathematical Institute and National Research University Higher School of Economics)

**Title:** Rationality problems in the higher-dimensional algebraic geometry

**Abstract:** The purpose of the talk is to survey old and new results on the rationality problem of algebraic varieties, mostly of dimension three.

**Speaker:** Armen Sergeev (Steklov Mathematical Institute of Russian Academy of Sciences)

**Title:** Quantization of the Theory of Topological Insulators

**Abstract:** Topological insulators are the solid bodies with a broad energy gap stable under small deformations. It motivates the usage of the topological methods in their investigation.

To quantize the theory of topological insulators we reformulate it in the language of  $K$ -theory. To do that we note that the algebra of observables of the topological insulators belongs to the class of graded  $C^*$ -algebras for which there is a variant of  $K$ -theory proposed by Van Daele. In terms of this theory it is possible to define also the topological invariants of insulators.

A key role in the investigation of the topological properties of solid bodies is played by the study of their symmetry groups. Kitaev has proposed a description of symmetries and classification of solid bodies based on the theory of Clifford algebras. In this way the quantization of the theory of topological insulators is reduced to the study of irreducible representations of Clifford algebras.

This quantization construction works in the bulk of the insulator, however at its boundary the energy gap which is necessary for the K-theory construction in the bulk may close at the boundary. To describe the arising gapless system we have to use another variant of K-theory proposed by Kasparov. In its terms it is possible to describe the algebra of the boundary observables and define the boundary topological invariants of insulators.

A relation between the topological invariants of the insulator and its boundary is established by the so called BB-correspondence. For its construction it is used a short exact sequence connecting the algebras of observables of the insulator and its boundary. It generates the long exact sequence of homomorphisms of K-groups such that the BB-correspondence coincides with the boundary map in this sequence.

**Speaker :** Andrey Shafarevich (Lomonosov Moscow State University, Steklov Mathematical Institute of Russian Academy of Sciences)

**Title:** Geometric asymptotic solutions of strictly hyperbolic systems with abruptly varying coefficients

**Abstract:** It is well known that short-wave asymptotic solutions of linear strictly hyperbolic systems with smooth coefficients are described in terms of certain geometric objects - Lagrangian manifolds or complex vector bundles over isotropic surfaces. These manifolds are invariant with respect to Hamiltonian fields whose Hamiltonians satisfy the characteristic equation for the leading symbol of the hyperbolic system.

If the coefficients are discontinuous, or depend singularly on a small parameter (i.e., their weak limits are not smooth), the solution has a more complex form near the support of the singularity; in general case, the corresponding theory is not developed. In the talk, we describe geometric asymptotics for the Cauchy problem in the case when the coefficients change abruptly, i.e. they or their weak limits are discontinuous on some hypersurface in the space of independent variables. In this situation, the Lagrangian manifolds and complex vector bundles change at points corresponding to the specified surface, and the variation is controlled by the geometry of the projective

hypersurface in the dual space, determined by the highest symbol of the system. It is proved that the solution can be expanded into an asymptotic series, the terms of which are expressed through the Maslov canonical operator on new invariant manifolds; the functions to which these operators are applied satisfy the auxiliary scattering problem for a linear system of ordinary differential equations, and the coefficients of the monodromy operator of such a problem determine the coefficients of reflection and transmission of waves through the support of discontinuity.

**Speaker:** Ye Tian (Academy of Mathematics and Systems Science, Chinese Academy of Sciences)

**Title:** Distribution on achiral manifolds

**Abstract:** For a fixed geometric model, we discuss the distribution of commensurable classes containing non-orientable members among commensurable classes containing achiral members, which is closely related to number theory. For Sol 3-manifolds, the distribution involves negative Pell equations, while for hyperbolic 3-manifolds, it involves strong approximation. The talk is based on our joint works with Shicheng Wang and Zhongzi Wang, and with Hang Yin.

**Speaker:** Xiangdong Ye (University of Science and Technology of China)

**Title:** Recurrence and its applications

**Abstract:** Recurrence is one of the most important properties in dynamical systems. It turns out that recurrence also has many profound applications in combinatorial number theory. In this survey talk I will review some known results concerning recurrence and the applications. In the process I will also explain the main tools used and under development, and state some recent results and open questions.

**Speaker:** Ping Zhang (Academy of Mathematics and Systems Science, Chinese Academy of Sciences)

**Title:** Global stability of large Fourier mode for 3-D anisotropic Navier-Stokes equations in cylindrical domain

**Abstract:** In this talk, we first present the global existence and stability of solutions to 3-D classical Navier-Stokes equations (*NS*) in an infinite cylindrical domain with large Fourier mode initial data. Then we extend similar result for 3-D anisotropic Navier-Stokes equations (*ANS*). We remark that due to the loss of vertical viscosity in (*ANS*), the construction of the energy functionals for (*ANS*) is much more subtle than that of(*NS*). Compared with our previous paper for (*NS*), we improve the polynomial decay in  $k$  for the Fourier coefficients of the solution to be exponential decay in  $k$  here.

**Speaker:** Weiping Zhang (Nankai University)

**Title:** Deformations of Dirac operators

**Abstract:** Dirac operators as well as their deformations have played important roles in many problems in geometry and topology. We will discuss some of these applications.

**Speaker:** Andrei Zotov (Steklov Mathematical Institute RAS)

**Title:** Integrable tops and 1+1 field theories via solutions of associative Yang-Baxter equation

**Abstract:** I will show how R-matrices satisfying the associative Yang-Baxter equation can be used for constructing Lax pairs with spectral parameter. First, we describe a class of models given by Liouville integrable finite-dimensional tops. Then we describe integrable 1+1 generalizations for these models. Finally, we discuss a relation of the constructed models to many-body integrable systems (and their field-theoretical generalizations).

## Abstracts of invited talks

**Speaker:** Dmitri Bykov (Steklov Mathematical Institute and Institute for Theoretical and Mathematical Physics (Moscow State University))

**Title:** The deformed  $CP^1$  sigma model in Gross-Neveu formalism

**Abstract:** I will review the recently established relation between 2D integrable sigma models (quantum theories of harmonic maps) and Gross-Neveu (GN) models - a class of models with quartic interaction. This framework can incorporate supersymmetric theories as well as deformed (i.e. non-homogeneous) target spaces. As an example of the latter, one can consider the deformed supersymmetric  $CP^1$  model, also known as the 'sausage'. I will show that this geometry naturally arises as the solution of Ricci flow equations of the deformed GN model, which incidentally are equivalent to Nahm's equations from the theory of monopoles. The conformal limit of this system is governed by the so-called super-Thirring model, whose correlation functions can be computed explicitly.

**Speaker:** Konstantin Fedorovskiy (Lomonosov Moscow State University)

**Title:**  $C^m$ -approximation of functions by solutions of second-order homogeneous elliptic equations on compact sets in  $\mathbb{R}^N$

**Abstract:** The capacity approximability criteria of the Vitushkin type will be presented for the problems on  $C^m$ -approximation of functions by solutions of homogeneous second-order elliptic partial differential equations  $Lf = 0$  with constant complex coefficients on compact sets in  $\mathbb{R}^N$  for the whole range of dimensions  $N \in \{2, 3, \dots\}$  and for the whole range of smoothness parameters  $m \in [0, 2)$ . These criteria are obtained in the case of approximation problem for individual functions, and they are stated in terms of  $C^m$ - $L$  and  $Lip^m$ - $L$ -capacities, which are special analytic characteristics of sets in  $\mathbb{R}^N$  related with the differential operator  $L$  that determines the equation by solutions of which the approximation is done. As an immediate corollaries of these results, the approximation criteria can be obtained for

classes of functions (they were previously known for various  $L$ ,  $N$  and  $m$  under consideration). Going further, we plan to present the results on  $C^m$ -approximation of functions by polynomial solutions of equations under consideration and by systems of such equations. Finally, new results about properties of the capacities mentioned above will be also presented. The talk is based on recent results by M. Mazalov, P. Paramonov and the author that were obtained in frameworks of the project 22-11-00071 by the Russian Science Foundation.

**Speaker:** Aleksandr Komlov (Steklov Mathematical Institute of Russian Academy of Sciences)

**Title:** The polynomial Hermite-Padé  $m$ -system and reconstruction of values of an algebraic function

**Abstract:** We introduce the polynomial Hermite-Padé  $m$ -system, which includes the Hermite-Padé polynomials of type I and type II and generalizes their constructions. We show that the polynomial Hermite-Padé  $m$ -system allows to reconstruct constructively the values of an algebraic function  $f$  of degree  $m+1$  on  $m$  sheets of its Riemann surface from a given germ of  $f$ .

**Speaker:** Mikhail Korobkov (Fudan University)

**Title:** On Ladyzhenskaya-Leray's Problem for a Flow of a Viscous Incompressible Fluid in a System of Channels

**Abstract:** The classical Ladyzhenskaya-Leray's problem concerning the stationary motion of a viscous incompressible fluid in a system of distorted infinite channels is studied under Dirichlet boundary conditions. In contrast to many previous works, the domain is not assumed to be simply-connected, and the fluxes are not assumed to be small. In this very general setting we prove, that the Leray invading domains method always generates a solution with the Dirichlet integral bounded uniformly in every bounded subdomain. This is a generalization of the classical Ladyzhenskaya-Solonnikov result obtained under the additional assumption of zero

boundary conditions. This is a joint result with Xiao Ren (Peking University) and Gianmarco Sperone (Politecnico di Milano).

**Speaker:** Vladimir Lysov (Keldysh Institute of Applied Mathematics)

**Title:** Classical properties of polynomials satisfying new interpolation conditions

**Abstract:** It is well known that orthogonal polynomials satisfy three-term recurrence relations, are denominators of Padé interpolations, determine the reproducing Christoffel-Darboux kernel, and solve some matrix Riemann-Hilbert problem.

Recently, biorthogonal Cauchy polynomials have been studied in connection with the theory of integrable systems. It turned out that these polynomials satisfy four-term recurrence relations, are associated with some interpolation problem, and have analogues of all the other properties listed above.

We introduce a new class of interpolation problems. For polynomials solving these problems, all the properties listed above are satisfied. In special cases, our construction leads to orthogonal polynomials and biorthogonal Cauchy polynomials.

**Speaker:** Hangyang Meng (Shanghai University)

**Title:** Coset complexes in finite groups

**Abstract:** In this talk, we will show some topological properties of proper coset posets in finite groups. Let  $G$  be a finite group and  $X$  be a subgroup of  $G$ . Denote by  $C_X(G)$  the set of all cosets  $Hx$  in  $G$  with  $X \leq H < G$ . We will show that  $C_X(G)$  is non-contractible if  $G$  is solvable or  $N_G(X)$  contains a Sylow 2-subgroup and a Sylow 3-subgroup of  $G$ . This result follows J. Shareshian and R. Woodroffe's work in Adv.Math(2016). We also give some divisibility properties of the Euler characteristic of  $C_X(G)$  when  $X$  is a  $p$ -group, which follows K. S. Brown's classical result in J. Algebra (2000).

**Speaker:** Pavel Mozolyako (St.-Petersburg State University)

**Title:** Hardy embeddings and Potential Theory on graphs

**Abstract:** Let  $\Gamma$  be a finite graph. Given a function  $K : V(\Gamma) \times V(\Gamma) \rightarrow \mathbb{R}_+$  and a

measure (a collection of non-negative numbers attached to vertices)  $\mu: V(\Gamma) \rightarrow \mathbb{R}_+$

we define the  $K$ -potential of  $\mu$  as follows

$$V_K^\mu(\alpha) = \sum_{\beta \in V(\Gamma)} K(\alpha, \beta) \mu(\beta).$$

We are interested in potential-theoretic properties of such an object for certain choices of  $\Gamma$ .

An important collection of examples is given by the *weighted Hardy potential* on directed acyclic graph which is defined to be

$$K_w(\alpha, \beta) = \sum_{\gamma \geq \alpha, \beta} w(\gamma),$$

where  $w: V(\Gamma) \rightarrow \mathbb{R}_+$  is a positive weight defined on vertices of  $\Gamma$ .

Such a scheme under a suitable choice of  $\Gamma$  appears in several well-known problems.

For instance, if  $\Gamma$  is a tree, then the model can be understood as the electrical network on a tree (and the weight  $w$  serves as the resistance).

If  $\Gamma$  is a Cartesian product of several branches (basically subsets  $[1, 2, \dots, M]$  of  $\mathbb{N}$ ), we have the weighted integration operator on  $\mathbb{R}_+^n$ .

If we choose  $\Gamma$  to be a product of several trees, the model has important connections to the weighted spaces of harmonic functions on the polydisc.

We will discuss such models and their relation to 'continuous' problems. In particular we are interested in maximum and domination principles, energy decay estimates and embedding theorems on products of trees.

**Speaker:** Wenhao Ou (Academy of Mathematics and Systems Science, Chinese Academy of Sciences)

**Title:** Orbifold modification of complex analytic varieties

**Abstract:** We prove that if  $X$  is a compact complex analytic variety, which has quotient singularities in codimension 2, then there is a projective bimeromorphic morphism  $f: Y \rightarrow X$ , such that  $Y$  has quotient singularities, and that the indeterminacy locus of  $f^{-1}$  has codimension at least 3 in  $X$ . As an application, we



deduce the Bogomolov-Gieseker inequality on orbifold Chern classes for stable reflexive coherent sheaves on compact Kaehler varieties which have quotient singularities in codimension 2.

**Speaker:** Liming Sun (Academy of Mathematics and Systems Science, Chinese Academy of Sciences)

**Title:** Attainability of the best constant of Hardy-Sobolev inequalities with full boundary singularities

**Abstract:** We consider a type of Hardy-Sobolev inequality, whose weight is singular on the whole domain boundary. We are concerned with the attainability of the best constant of such inequality. In dimension two, we link it to a conformally invariant one using the conformal radius of the domain. The best constant of such inequality on a smooth domain is achieved if and only if the domain is non-convex. In higher dimensions, the best constant is achieved if the domain has negative mean curvature somewhere.

**Speaker:** Dongmeng Xi (Shanghai University)

**Title:** Chord measures in Integral Geometry and their Minkowski problems

**Abstract:** We introduced a new family of translation invariant geometric measures arising from Integral Geometry. These measures are related to a family of Monge-Ampère type operators converging to a  $\sigma_k$  operator. Their Minkowski type problems are proposed, and are solved except for the critical case. This talk is based on a joint work with Erwin Lutwak, Deane Yang, and Gaoyong Zhang.

**Speaker:** Daxin Xu (Academy of Mathematics and Systems Science, Chinese Academy of Sciences)

**Title :** Exponential sums, differential equations and geometric Langlands correspondence

**Abstract:** In 1970s, Dwork established a relationship between the Bessel differential equation and the Kloosterman sums. Such a relationship can be regarded as an

instance of the geometric Langlands correspondence for  $GL_2$ . In this talk, we will first review some classical results on exponential sums and differential equations, and then discuss some recent progress on generalizations of Dwork's result from the perspective of geometric Langlands correspondence. It is based on joint works with Xinwen Zhu and Kamgarpour-Yi.

**Speaker:** Alexander Zheglov (Lomonosov Moscow State University)

**Title:** Normal forms for ordinary differential operators

**Abstract:** Considering the ring of ordinary differential operators  $D_1 = K[[x]][d]$  as a subring of a certain complete non-commutative ring  $D_1$  (not the known ring of formal pseudo-differential operators!), the normal forms of differential operators mentioned in the title are obtained after conjugation by some invertible operator («Schur operator»), calculated using one of the operators in a ring. Normal forms of commuting operators are polynomials with constant coefficients in the differentiation, integration and shift operators, which have a finite order in each variable, and can be effectively calculated for any given commuting operators.

According to the well known classification (Krichever's theorem and its various generalisations), any commutative subring of ODOs can be encoded in terms of spectral data, consisting of a projective curve (may be singular), a spectral sheaf of rank  $r$  (a torsion free sheaf with chosen basis of  $r$  global sections and vanishing first cohomologies), and some additional formal data.

I'll talk about some applications of the theory of normal forms: an effective parametrisation of spectral sheaves on a spectral curve, and an alternative way to find examples of commuting differential operators of arbitrary rank. It is expected that the study of normal forms of commuting operators can subsequently help in the description of the moduli space of torsion free sheaves with a fixed Hilbert polynomial on the spectral manifold of a ring of commuting operators of arbitrary dimension, as well as in solving the problem of finding explicit examples of difference or differential-difference commuting operators.

The talk is based on a joint work with Phd student Junhu Guo.