

Structures on Manifolds (program of the course)

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0. Basic concepts. Ringed spaces. Compact manifolds. Structure sheafs on manifolds of various types. Forgetting functors.

1. Elements of manifolds topology. Orientation. Connected sums, their relation with blow-ups in the case of algebraic surfaces. Overview of the cohomological theories. One-dimensional manifolds.

2. Compact topological surfaces. The smooth structure and its uniqueness. Coincidence of the almost complex and complex structures. Complete topological classification of surfaces. Moduli spaces of algebraic curves.

3. Smooth structures. Smoothing of the 3-manifolds; the uniqueness of the smooth structure on them. Examples of non-smoothable manifolds. The sphere \mathbf{S}^7 and the *group* of smooth structures on it.

4. Almost complex structures. Definitions. Examples of the even-dimensional manifolds admitting no almost complex structures; the sphere \mathbf{S}^4 . The geography of almost complex 4-dimensional manifolds.

5. Complex structures. Integrability of almost complex structures. The examples of non-integrable almost complex structures. The sphere \mathbf{S}^6 , almost complex structure on it and the problem of existence of a complex structure.

6. Kahler metrics on complex varieties. The relation between symplectic, kahler and almost complex structures. The products of odd-dimensional spheres as complex manifolds and the possibility of introduction the kahler structures on them; the Hopf surface $\mathbf{S}^1 \times \mathbf{S}^3$.

7. Complex and algebraic varieties. The complex varieties, admitting and not-accepting an algebraic structure. Complex tori and abelian varieties. K3-surfaces.

8. Various problems. География алгебраических поверхностей. Алгебраические и симплектические многообразия: параллели, зеркальная симметрия. Множество римановых метрик на гладком многообразии; пространства Тайхмюллера, потоки Риччи и Кэлера-Риччи. Гипотеза геометризации Терстона и её доказательство Перельманом; гипотетические параллели с программой минимальных моделей.