

STEKLOV INTERNATIONAL MATHEMATICAL CENTER

STEKLOV MATHEMATICAL INSTITUTE
OF RUSSIAN ACADEMY OF SCIENCES

EDUCATIONAL CENTER

Fall semester 2023/2024

«Elements of Optimal Control Theory»

(Yuri L. Sachkov)

A course of 10-12 lectures will be devoted to the main issues of the theory of optimal control: the existence of solutions and the necessary optimality conditions, with a presentation of the required mathematical foundation. The course focuses on two major results of the theory:

- Sufficient Filippov conditions for the existence of an optimal control;
- Pontryagin's maximum principle.

Both results will be presented and proved in an invariant geometric form for problems on smooth manifolds.

Along the way, important general mathematical material will be presented:

- Measurable sets and functions, Carathéodory differential equations;
- Differential equations on smooth manifolds;
- Elements of chronological calculation R.V. Gamkrelidze-A.A. Agracheva;
- Differential forms;
- Elements of symplectic geometry.

The course is designed for students of mathematics and physics (starting from the 3rd year) and graduate students. No prior knowledge of control theory is assumed.

The course is a continuation and deepening of the online course given by Yu.L. Sachkov in 2022, but it can also be listened to independently. At the end of the course, an exam is planned for those who wish.

Course program

1. Statement of the optimal control problem.
2. Measurable sets and functions, Carathéodory differential equations.
3. Sufficient Filippov conditions for the existence of an optimal control.

4. Differential equations on smooth manifolds.
5. Elements of chronological calculus R.V. Gamkrelidze-A.A. Agrachev.
6. Differential forms.
7. Elements of symplectic geometry.
8. Proof of the Pontryagin maximum principle on manifolds: geometric form, optimal control problems with different boundary conditions.

References

- [1] *A.A. Agrachev, Yu.L. Sachkov, Geometric control theory. M.: Fizmatlit, 2005. Engl. trans.: A.A. Agrachev, Yu.L. Sachkov, Control theory from the geometric viewpoint. Springer-Verlag. 2004*
- [2] *A.A. Agrachev, D. Barilari, U. Boscain, A Comprehensive Introduction to sub-Riemannian Geometry from Hamiltonian viewpoint. Cambr. Stud. in Advanced Math., Cambr. Univ. Press, 2019.*
- [3] *Yu.L. Sachkov, Introduction to geometric control theory. M.: URSS, 2021, 160 pp. Engl. trans.: Yu. Sachkov, Introduction to geometric control. Springer Verlag, 2022.*