

STEKLOV INTERNATIONAL MATHEMATICAL CENTER

STEKLOV MATHEMATICAL INSTITUTE  
OF RUSSIAN ACADEMY OF SCIENCES

EDUCATIONAL CENTER

Fall semester 2023/2024

«**Elements of Dynamical Systems**»

(Oleg È. Zubelevich)

The aim of the lecture course is to show the diversity of problems and methods of dynamical systems. The lecture course consists of several independent topics. We consider finite and infinite-dimensional dynamical systems and discuss the classical results as well as the results obtained in the last twenty years.

1. Dynamical systems with discrete time: the Poincare mapping at an energy level in a Hamiltonian system; other examples. Invariant measure, the ergodic theorems of Yosida, von Neumann, and Birkhoff. The Poincare recurrence theorem. Introduction to ergodic systems: basic theorems, the Koopman operator. Ergodicity of the tent map, ergodicity of the shift in the torus.
2. Other causes of the dynamical chaos' appearance: separatrix splitting. The Poincare- Melnikov integral.
3. Discret Lagrangian systems: the anti-integrable limit.
4. Time averaging in another context: an infinite-dimensional version of the Massera theorem and its application to elasticity theory.
5. Bounded solutions to the systems of the second order ODE.
6. A phase flow of an ODE with a smooth right-hand side:  $\omega$ -limit set and its properties, attractors.
7. ODE with discontinuous right-hand side, differential inclusions Filippov's regularization, periodic solutions to systems with dry friction.

References

- [1] *V. Arnold*, Ordinary Differential Equations. Springer, 1992.
- [2] *R. Temam*, Infinite-dimensional dynamical systems. Springer, 1993.
- [3] *D. Treschev, O. Zubelevich*, Introduction to the Perturbation Theory of Hamiltonian Systems. Springer, 2010.
- [4] *A.F. Filippov*, Differential Equations with Discontinuous Right-Hand Sides, Transaction of A.M.S., 42 (1964), pp. 199-231.