STEKLOV INTERNATIONAL MATHEMATICAL CENTER

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«Neighbourhood semantics of modal logics»

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Neighbourhood semantics is a natural generalization of Kripke semantics and topological semantics. Despite this, discussion of neighbourhood semantics is usually left out of basic courses of modal logic. However, a number of modal systems (primarily non-normal ones) are incomplete with respect to the more common Kripke semantics, but complete in the neighbourhood case. Moreover, there are important modal systems (for example, the Gödel-Löb provability logic GL) that are strongly complete in the neighbourhood case, but not in the case of Kripke. In our course, we plan to tell the main theorems and facts about neighbourhood semantics, analyze specific examples of interesting neighborhood-complete logics, and also talk about normal logics for which neighborhood semantics gives interesting non-trivial results. Students are required to have a good knowledge of classical propositional logic. Familiarity with modal logic is desirable, but not required.

Course plan:

1. Basic definitions: neighbourhood semantics, Kripke semantics, topological semantics.

2. Definable properties of neighbourhood frames, bisimulations, truth-preserving operations.

3. Normal and non-normal modal logics. A general soundness theorem.

4. Construction of canonical models. Completeness for the logics E, EC, EN, EM, K.

5. Filtrations and decidability of modal logics.

6. The standard translation into first-order logic.

7. The logic $S4$ and its neighbourhood frames as topological spaces. Extensions of the logic $wK4$ and derivational semantics as a special case of neighbourhood semantics.

8. Neighborhood semantics of modal logics GL and $S4CI$.

9. Construction of a neighbourhood frame from a Kripke frame (construction of paths with stops).

10. Products of Kripke frames and neighborhood frames. Axiomatization and the completeness theorem for products of logics from a set $\{D, T, D4, S4\}$.

11. Axiomatization and the completeness theorem for $K \times K$.