STEKLOV INTERNATIONAL MATHEMATICAL CENTER

STEKLOV MATHEMATICAL INSTITUTE
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EDUCATIONAL CENTER

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«Complex cobordisms»

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The standard homology groups $H_*(X;\ Z)$ contain the information about the «independent cycles» inside the space X. This idea is usually formalized by considering linear combinations of continuous maps from simplices into the space, and cycles correspond to such linear combinations that actually define a continuous map to X from the complex «without boundary» glued from the simplices. Moreover, two such cycles are homologous if their union is a boundary of some similar complex glued from simplices of greater by one dimension.

However, historically the idea of «cycles» is due to A. Poincaré, who originally considered smooth closed manifolds as «cycles». In such case two smooth cycles are homologous if their union is a boundary of a smooth manifold of greater by one dimension. But there were some technical reasons to use simplices complexes instead of smooth manifolds, primarily because «homology» defined in terms of smooth manifolds turned out to be much more complecated than more familiar for us «simplicial» one. Even such «homology» groups of point are far from trivial. However, it turns out that in this way one can still define a true (extraordinary) (co)homology theory, which is called the (co)bordism theory. Appearing in the middle of the 20th century, this theory developed rapidly in its second half.

A special place among all cobordism theories is occupied by complex cobordism, probably the most studied and most important cobordism theory. The course will cover both classical results on complex cobordism, such as the Milnor-Novikov theorem, Quillen's theorem on the universality of the formal group law of geometric cobordism introduced in [2], Landweber's exact functor theorem, and more modern results concerning equivariant complex cobordism, mainly with torus action.

1. Extraordinary (co)homology theories, Brown's representability theorem, spectra.

2. (Co)bordism theories, geometric definition, homotopy definition in terms of Thom spectra, their equivalence, Pontryagin-Thom theorem.

3. Adams Spectral Sequence, structure results on the coefficient ring of complex cobordism, Milnor-Novikov theorem.

4. Complex oriented cohomology theories, Chern classes, complex cobordism as universal complex oriented cohomology theory.

5. Formal group laws and their connection with complex cobordism, Quillen's theorem on the universality of the formal group law of complex cobordism.

6. Hirzebruch genera, Landweber's Exact Functor Theorem.

7. Equivariant (co)bordisms, difference between geometric and homotopy approaches in this case.

8. Equivariant cobordism with torus action, universal toric genus, localization formulae, equivariant and rigid genera.

Additional topics

(will be covered only if time permits)

9. Brown-Peterson spectra.

10. Adams-Novikov Spectral Sequence and chromatic approach to stable homotopy theory.

11. Structure results in equivariant complex cobordism, equivariant Quillen's theorem.

Recommended literature:

[1] *R. Stong*, Notes on Cobordism Theory. Princeton University Press, 1968.

[2] *S.P. Novikov*, The methods of algebraic topology from the viewpoint of cobordism theory. Math. USSR-Izv., 1:4 (1967), 827-913.

[3] *J.F. Adams*, Stable Homotopy and Generalized Homology, University of Chicago Press, 1974.