Spaces of functions integrable by finite additive invariant measures and representations of groups

V.Zh. Sakbaev

We study measures on a separable Hilbert space that are invariant with respect to such transformation groups as the group of shifts, the group of orthogonal transformations, the group of shifts along vector fields [1].

Unitary representations of these groups in the space of functions that are quadratically integrable by invariant measure are obtained. Subgroups whose representations are continuous in a strong operator topology are found.

For compositions of independent random transformations, conditions are obtained that are sufficient to fulfill and violate the law of large numbers. Averaging of random transformations are described by dissipative semigroups of contractions. Generators of these semigroups are investigated by means of Sobolev spaces and spaces of smooth functions.

[1] Sakbaev V.Zh. Flows in Infinite-Dimensional Phase Space Equipped with a Finitely-Additive Invariant Measures. Mathematics, 2023, **11** (5), 1161.

On the geometric property of smooth cylinder maps close in the C^1-norm to skew products and its dynamical applications

L.S. Efremova

C^1-smooth self-maps of a compact cylinder close to C^1-smooth skew products (and satisfying additional conditions of the Ω-stability or structural stability of the quotient of an unperturbed skew product) are considered.

The existence is proved of C^1-smooth invariant local lamination. With the use of this property the geometric integrability of maps under consideration is established.

With the use of the obtained results the example of the family of C^1-smooth maps close in the C^1-norm to skew products is constructed so that each map of this family admits a global chaotic attractor with a complicated topological structure of a ramified continuum.

The concept of dense intermittency (in the compliment of the attractor) of different ω-limit sets, the union of which coincides with the constructed attractor, is introduced.

The Koopman representation of Hamiltonian flows in infinite dimensional spaces with invariant measure

V.A. Glazatov

According to A. Weil's theorem, there is no infinite-dimensional version of the Lebesgue measure, and therefore the question arises of the need to construct a similar construction, albeit with the loss of some properties of the original measure.

The construction of analogs of the Lebesgue measure on infinite-dimensional locally convex spaces is required to study the quantization procedure for infinite-dimensional Hamiltonian systems (in particular, second quantization), for problems of statistical mechanics, for studying random unitary groups and the dynamics of open quantum systems.

The report tells about the work begun in [1], where the problem was posed of studying measures on an infinite-dimensional symplectic space that are invariant under the symplectomorphism group.

In [1], to obtain the result, the restrictions imposed by A. Weyl's theorem were relaxed, due to which the desired measure was obtained - an extended translation invariant measure from the papers [2] and [3] to a measure invariant with respect to the subgroup of the group of symplectomorphisms of the Euclidean phase space, which leave the two-dimensional symplectic subspaces of the phase space invariant. Such a measure was called a symplectic measure.

The talk will present the approach given in [1], by which the solutions of the Hamiltonian equations admitting singularities [4] can be described by means of a phase flow in the extended phase space and the unitary group corresponding to the Koopman representation. Thanks to this, the generator of the Koopman group will be investigated using the example of the Hamiltonian of a countable set of noninteracting harmonic oscillators.

[1] Glazatov V.A., Sakbaev V.Zh. Measures on a Hilbert space that are invariant under Hamiltonian flows. // Ufa Math. J. 2022. V.14, No. 2. pp. 3-21 (2022)

[2] Sakbaev V.Zh. Averaging of random walks and shift-invariant measures on a Hilbert space. Theor. Math. Phys. V. 191, No. 3. pp. 886-909 (2017)

[3] Busovikov V.M. Properties of a finitely additive measure on l_p that is invariant under shifts. Proceedings of the Moscow Institute of Physics and Technology. T.10, No. 2. C. 163-172 (2018)

[4] Vlasov S.N., Talanov V.I. Distributed wave collapse in the model of the nonlinear Schr?dinger equation. On Sat. Nonlinear waves. Dynamics and evolution. 1989. M: Science.

Higher order traps in quantum control landscapes

B.O. Volkov

Traps in quantum control landscapes are controls that complicate the search of globally optimal controls of the quantum objective functional by local search optimization methods. The notion of the higher order trap was previously introduced in [A. N. Pechen, D. J. Tannor, B"Are there traps in quantum control landscapes?B", Phys. Rev. Lett., 106 (2011), 120402], where traps of the 3rd order were found. We show that traps of arbitrarily high order exist for some controlled quantum systems.

Approximation of solutions for evolution equations by Feynman-Chernoff iterations

R.Sh. Kalmetev

In this talk we study averages of Feynman-Chernoff iterations[1] for random strongly continuous operator-valued functions whose values are bounded linear operators on a separable Hilbert space.

Let $\mathcal{H}$ be a separable Hilbert space, $\{F_k(t)\}, k \in \mathbb{N}$ be a sequence of independent identically distributed random strongly continuous operator-valued functions $\Omega \to C_s(\mathbb{R}_+, B(\mathcal{H}))$ defined on the probability space $(\Omega, \mathcal{A}, \mu)$. Let also the means $\mathbb{E}F_k(t) : \mathbb{R}_+ \to B(\mathcal{H})$ be Chernoff equivalent [1] to some semigroup. Then, by Chernoff's theorem, for sufficiently large N the finite composition of operators

$$\prod_{i=1}^{N} \mathbb{E}F_i\left(\frac{t}{N}\right) = \mathbb{E}\left(\prod_{i=1}^{N} F_i\left(\frac{t}{N}\right)\right),$$

called Feynman-Chernoff iterations, can be used to approximate solutions of the corresponding Kolmogorov equation.

Moreover, in the case when the values of operator-valued functions belong to the representation of some finite-dimensional Lie group, the algorithm for constructing an approximation has a lower computational complexity compared to the standard Monte Carlo algorithm which uses the Feynman-Kac formula. In particular, in this talk we consider in detail the case of the group of affine transformations of the Euclidean space [2].

[1] Orlov Yu.N., Sakbaev V.Zh., Smolyanov O.G. Unbounded random operators and Feynman formulae// Izvestiya: Mathematics, 80(6) , 1131B–1158 (2016).

[2] Kalmetev R.Sh., Orlov Yu.N., Sakbaev V.Zh. Chernoff Iterations as an Averaging Method for Random Affine Transformations// Comput. Math. and Math. Phys., 62, 996-1006 (2022).

Control landscape of measurement-assisted transition probability for a three-level quantum system with dynamical symmetry

M.A. Elovenkova

Quantum systems with dynamical symmetries have conserved quantities which are preserved under coherent control evolution and hence such systems can not be completely controlled by only coherent control. Quantum measurements allow to break to some degree the dynamical symmetry. Measurement-assisted coherent control of a simplest non-trivial three-level quantum system with dynamical symmetry was partially studied in [1], where maximum attained probability between the ground and the intermediate state was computed. In this talk, we present a detailed analysis of this case, providing complete description of the kinematic control landscape, where we find in addition to global maxima and minima also saddles and second-order traps [2].

[1] F. Shuang, M. Zhou, A. Pechen, R. Wu, O.M. Shir, and H. Rabitz, Control of quantum dynamics by optimized measurements, Phys. Rev. A 78, 063422 (2008).

[2] M. Elovenkova and A. Pechen, Control landscape of measurement-assisted transition probability for a three-level quantum system with dynamical symmetry (in preparation).

Gradient optimization and exact dynamics for two-level open quantum systems driven by coherent and oincoherent controls

V.N. Petruhanov

We consider the state-to-state transfer control problem for an open two-level quantum system (qubit) with evolution driven by coherent and incoherent controls and governed by master equation with dissipators of the GKSL-type in the weak coupling limit. We derive exact expressions for the dynamics of the system and for the gradient of the objective functional and study different regimes of the dynamics for the two-dimensional control space.

Uncertainty relations in terms of generalized entropies from information diagrams

A.E. Rastegin

Measurements of special kind are indispensable in quantum information science. For many of them we can estimate the index of coincidence. Using uncertainty relations to test non-classical

correlations, we try to make the corresponding restrictions as tight as possible. Inequalities between entropies and the index of coincidence are significant in information theory. The so-called information diagrams provide a tool to get such inequalities. In the literature, the considered question is discussed in application to the Shannon entropy. At the same time, Rényi and Tsallis entropies have found use in various disciplines. This talk is devoted to entropic uncertainty relations derived from information diagrams. The obtained inequalities are applied to mutually unbiased bases, mutually unbiased measurements and equiangular tight frames.

Entanglement as a resource for ensemble coherence and mutual information

A.D. Kodukhov

Observable applied to a part of an entangled state produces an ensemble of quantum states in the second subsystem and consumes entanglement measure. The approach is a generalization of the wave-particle duality principle in the Young interference experiment. We build an inequality connecting basis-free ensemble coherence, entanglement and observableB's uncertainty. We also show that the entanglement consumption coincides with the Holevo quantity which upper-bounds path-information in the interference experiment. This is joint work with D.A. Kronberg.

Master equations for composite open quantum systems

A.E. Teretenkov

Usually, the theory of open quantum systems considers the dynamics of the system in the reservoir assuming that the whole dynamics of the system and the tank is unitary. However, in this paper it will be assumed that the quantum dynamics is initially considered to be the one of the open composite system, and we will be interested in the dynamics of one of subsystems. There is an interest in this setup both in terms of recent physical applications and because it leads to non-trivial results already in the case of the finite-dimensional Hilbert space of the composite system. Purely algebraic formulas for the perturbation series terms defining the time-convolutionless master equations for the dynamics of a subsystem of such a composite open system will be given. We will also formulate the conditions under which the Bogolubov-van Hove limit exists, and discuss some corrections to this limit. An example illustrating our results will be also given.

On optimization of coherent and incoherent controls in one- and two-qubit open systems

O.V. Morzhin

We consider some one- and two-qubit open systems determined by the Gorini–Kossakowski–Sudarshan–Lindblad master equation with coherent (entering in the Hamiltonians) and incoherent (entering in the superoperator of dissipation and, possibly, in the Hamiltonians via the Lamb shift) open-loop controls. We study optimal control problems with various classes of controls, objective functionals (e.g., for minimizing the Hilbert–Schmidt distance between final and target density matrices), and constraints, describe various results obtained by adapting various tools from the theory of optimal control and finite-dimensional optimization (Krotov, gradient projection methods, stochastic optimization, etc.). These optimal control problems and the corresponding

results are considered following the general course for analyzing various pitfalls and possibilities of the mathematical theory of quantum optimal control.

Statistical structure of quantum theory and quantum information
A.S. Holevo

Incoherent control of open quantum systems
A.N. Pechen

In this talk, we will discuss two approaches for using environment for controlling quantum systems. One approach is to use state of the environment to manipulate the controlled system. Another approach is to use back-action of non-selective quantum measurements as an external action to control the system. In both cases, for various quantum control problems an additional to coherent control degree of controllability can be attained.

Spaces of Cauchy transforms and localization of zeros
A.D. Baranov

The class of functions representable as the Cauchy integrals with L^2 data with respect to some fixed measure, forms a Reproducing Kernel Hilbert space which consists of functions analytic outside the support of the measure. Such spaces naturally appear in functional models for rank one perturbations of normal operators. In the talk we concentrate on the case of discrete measures in the complex plane with the following property: up to a finite number, all zeros of any Cauchy transform of the measure are localized near the support of the measure. We find several equivalent forms of this property and show that the parts of the support attracting zeros of Cauchy transforms are ordered by inclusion modulo finite sets. The talk is based on joint works with E. Abakumov and Yu. Belov.

Representing systems of reproducing kernels in spaces of analytic functions
T.G. Batenev

In the talk we give an elementary construction of reproducing kernels for Hardy spaces H^p, $p \in (1, \infty)$, in the disk, polydisc, ball and in the half-plane, as well as representing systems of reproducing kernels in some weighted Hardy spaces in disc (e.g., the Dirichlet space).

Weighted Hardy Type Embeddings on Graphs
P.A. Mozolyako

Controllability of some closed quantum systems
S.A. Kuznetsov

This talk will address the controllability problem of several specific closed quantum systems. Namely, it will briefly observe properties of their dynamical Lie algebras, which are constructed using corresponding Hamiltonians and reveal their - systems - behavior under a wide range of control classes. This observation will be held in terms of the necessary controllability condition

and the controllability criteria applied to some systems, which are relevant for applications and theoretical analysis.

Analysis of traps in quantum control landscapes using gradient methods

A.A. Miachkova

An important direction in the development of quantum technologies is the study of control of atomic and molecular systems [1-3]. Analysis of the presence or absence of traps, including n-order traps, is important for determining the correct algorithms for finding optimal control fields [4]. In this talk, we discuss our results on numerical experiments to study the control landscapes for various quantum systems including three-level systems with traps of the orders 3 and 7 and four-level systems [5]. This is joint work with A.N. Pechen.

[1] S.A. Rice and M. Zhao, Optical control of molecular dynamics, John Wiley & Sons, Inc., 2000.

[2] V.S. Letokhov, Laser control of atoms and molecules, Oxford Univ. Press, 2007.

[3] D.J. Tannor, Introduction to quantum mechanics: A time dependent perspective. Sausalito, University Science Books, 2007.

[4] A.N. Pechen and D.J. Tannor, Quantum control landscape for a lambda-atom in the vicinity of second-order traps, Israel Journal of Chemistry, 52(5):467-472, 2012.

[5] B.O. Volkov, A.A. Miachkova, and A.N. Pechen, Higher order traps in quantum control landscapes (in preparation).

Machine learning of quantum control landscapes

N.S. Fedorets

Machine learning is widely used for finding suitable controls for maximizing objectives for quantum systems. In this work, we discuss possible use of neural networks for the search of traps in quantum control landscapes.

E.A. Lopatin

On the application of the methods of potential theory on compact Riemann surfaces to the description of the weak asymptotic behavior of Hermite-Padé polynomials

On the Laplace method for functions of large numbers: some improvements

A.A. Imomov

The report presents an improved version of Laplace's theorem on the asymptotic expansion of the Riemann integral of a function of large numbers. The asymptotic formula obtained in our theorem generalizes and improves, pointing to the estimate of the remainder terms, a number of asymptotic formulas of classical analysis, such as the formulas of Stirling, Legendre, Wallis, etc.

On Kantorovich' non-linear tasks of optimal measure transport

S.N. Popova

Random walks on a line and algebraic curves
S.V. Grishin

On constructing of informationally complete operator-valued measures
G.G. Amosov

On C*-algebra and $*$-polynomial relations
R.N. Gumerov

Categorical constructions arise in the natural way in the theory of universal C^*-algebras generated by sets of generators subject to certain relations. In this talk, we discuss properties of the categories introduced by T.A. Loring and called C^*-relations as well as properties of certain functors between these categories. Objects of C^*-relations are functions from sets to C^*-algebras and morphisms are $*$-homomorphisms between C^*-algebras making the appropriate triangles commute.

The talk is based on the results of joint work with Berdnikov I.S., Lipacheva E.V. and Shishkin K.A. (Kazan Federal University).

Extensions of conditionally completely positive maps between operator systems
V.I. Yashin

In noncommutative probability theory, conditionally completely positive maps arise as the generators of completely positive semigroups and in the study of Levy processes. In the talk, we will propose a definition for conditionally completely positive maps between operator systems and in the finite dimensional case we prove the Arveson-type theorem on the injectivity of matrix algebras.

On semigroup C^*-algebra generated by a semidirect product of semigroups
E.V. Lipacheva
On a semigroup C^*-algebra generated by a semidirect product of semigroups

The report deals with the C^*-algebra generated by the regular representation of the semidirect product of semigroups $S \rtimes P$. We prove that this C^*-algebra is isomorphic to the reduced crossed product of the semigroup C^*-algebra $C_r^*(S)$ by the semigroup P provided that P acts on the semigroup S by automorphisms. The case when P is a group was considered in [1]. We apply these results to the study of the structure of the C^*-algebra generated by the regular representation of the semidirect product $Z \rtimes Z^\times$.

[1] Lipacheva E.V. On a semigroup C^*-algebra for a semidirect product. Lobachevskii J. Math. **44** (7) (2023) (in press).

Extension of the family of projections to a positive operator-valued measure and restoration of the quantum state after measurement.
A.O. Alekseyev

Covariance requirement for calculating the capacity of quantum channels introduced by projective unitary irreducible representations of finite groups.

L.A. Ryskin

On the symbols of mixed unitary quantum channels generated by finite unitary groups.

D.D. Cheremukhin

On generating channels

R.L. Khazhin

The report deals with quantum channels for composite quantum systems. We consider the quantum channels which uniquely define the channels for subsystems. Such channels for composite systems are said to be generating. We discuss properties and give several examples of the generating quantum channels.