# International conference dedicated to the 100th anniversary of the birthday of V.S. Vladimirov (Vladimirov-100)

January 9–14, 2023, online

НАУКА И УНИВЕРСИТЕТЫ
НАЦИОНАЛЬНЫЕ ПРОЕКТЫ РОССИИ

SIMC
Steklov International Mathematical Center

МИАН

*Steklov Mathematical Institute*
*of Russian Academy of Sciences, Moscow*
*Steklov International Mathematical Center, Moscow*

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# 9 January, Monday

## 10:45–11:00 (MSK), 08:45–09:00 (CET)

Opening

## 11:00–11:30 (MSK), 09:00–09:30 (CET)

**I.V. Volovich**
*Steklov Mathematical Institute, Russia*
**Title:** p-adic Mathematical Physics and V.S.Vladimirov Works
**Abstract:** After a brief reminder of the basic principles of p-adic mathematical physics, we will discuss recent developments in the quantum zeta function. V.S. Vladimirov is one of the founders of p-adic mathematical physics. His works on spectral theory in p-adic quantum mechanics, fractional differential operator and non-linear pseudo-differential equations will be discussed in the context of recent developments.

## 11:30–12:00 (MSK), 09:30–10:00 (CET)

**Vladimir Anashin**
*Faculty of Computational Mathematics and Cybernetics, Lomonosov Moscow State University, Russia &* *Federal Research Center `Information and Control' , Russian Academy of Sciences, Russia*

**Title:** Взгляд на квантовые системы сквозь призму p-адических 1-липшицевых функций
A look at quantum systems via p-adic 1-Lipschitz maps

**Abstract:** В докладе будет рассказано о том, что основные понятия квантовой теории (волновая функция, наблюдаемые, чистые состояния, смешанные состояния и т.п.), могут быть естественным образом выражены в терминах функций, чья область определения и область значений есть кольцо целых p-адических чисел. Эти функции представляют собой класс всех каузальных функций; последние, в свою очередь, могут рассматриваться как описания эволюции квантовых систем в теориях с минимальными длинами. В этих терминах оказывается возможным описать, каким образом возникает случайность в квантовых системах, коллапс волновой функции, и т.п. Данный подход мотивирован идеями И.В.Воловича и Г. эт Хоофта и может рассматриваться как некоторый вклад в p-адическую математическую физику, развивать которую начал В.С.Владимиров в 1988 году. Доклад в основном базируется на статье Vladimir Anashin. `Toward the (non-cellular) automata interpretation of quantum mechanics: Volovich postulates as a roadmap', International Journal of Modern Physics A, Vol. 37, No. 20--21, 2243003 (2022).

In the talk, it will be argued that basic notions of quantum theory (wave function, observable, pure state, mixed state, etc.) can naturally be expressed in terms of functions whose

domain and range are p-adic integers and which satisfy the p-adic Lipschitz condition with a constant 1 (briefly, the p-adic 1-Lipschitz maps). The latter maps constitute the class of all causal functions; the functions describe evolution of quantum systems in theories with a minimum scale lengths. In these terms it is possible to reveal how randomness emerges in quantum systems, how wave function collapses, etc. The approach is motivated by the ideas of I.V.Volovich and G.`t Hooft and can be judged as a contribution to the p-adic mathematical physics initiated by V.S.Vladimirov in 1988. The talk is mostly based on the paper `Toward the (non-cellular) automata interpretation of quantum mechanics: Volovich postulates as a roadmap', International Journal of Modern Physics A, Vol. 37, No. 20--21, 2243003 (2022) by Vladimir Anashin.

## 12:00–12:30 (MSK), 10:00–10:30 (CET)

**David Weisbart**
*University of California, Riverside, USA*
**Title:** Components and Exit Times of Brownian Motion in two or more p-Adic Dimensions
**Abstract:** The fundamental solution of a pseudo-differential equation for functions defined on the $d$-fold product of the $p$-adic numbers, $\mathds{Q}_p$, induces an analogue of the Wiener process in $\mathds{Q}_p^d$. As in the real setting, the components are $1$-dimensional $p$-adic Brownian motions with the same diffusion constant and exponent as the original process. Asymptotic analysis of the conditional probabilities shows that the vector components are dependent for all time. Exit time probabilities for the higher dimensional processes reveal a concrete effect of the component dependency.

## 13:00–13:30 (MSK), 11:00–11:30 (CET)

**Branko Dragovich**
*Institute of Physics, University of Belgrade, Belgrade, Serbia & Mathematical Institute, Serbian Academy of Sciences and Arts, Belgrade, Serbia*
**Title:** From p-adic to zeta scalar particles
**Abstract:** In this talk, we start from the Lagrangian for p-adic open strings, which is slightly modified so that it does not contain tachyons and ghosts. This Lagrangian makes sense not only for prime numbers but also for all natural numbers. Taking a suitable sum of these Lagrangians over all natural numbers and using Euler's formula for the Riemann zeta function, a new Lagrangian is obtained. This new Lagrangian contains non-locality through a non-local operator in the form of the Riemann zeta function of the d'Alembertian. We investigate the equations of motion of the corresponding scalar field, especially in the weak field approximation. We show that this scalar field can exist in a closed and an open FLRW universe with a cosmological constant.

## 13:30–14:00 (MSK), 11:30–12:00 (CET)

**E.I. Zelenov**
*Steklov Mathematical Institute, Russia*
**Title:** On geometry of p-adic coherent states and mutually unbiased bases
**Abstract:** TBA

## 14:00–14:30 (MSK), 12:00–12:30 (CET)

**Patrick Erik Bradley**
*Karlsruhe Institute of Technology, Germany*
**Title:** Locally Vladimirov operators on the p-adic points of a Mumford curve
**Abstract:** After an introduction to Mumford curves as a kind of p-adic analogue of Riemann surfaces, new psedodifferential operators generalising the Vladimirov operator are introduced. They induce a wavelet decomposition of the Hilbert space of functions on the p-adic rational points on a Mumford curve. The Cauchy problem for the heat equation on those points of Mumford curves, in a special case is also addressed.

## 15:30–16:00 (MSK), 13:30–14:00 (CET)

**V.V. Zharinov**
*Steklov Mathematical Institute, Russia*
**Title:** Симметрии и законы сохранения уравнения Лиувилля
Symmetries and conservation laws of the Liouville equation
**Abstract:** В рамках алгебро-геометрического подхода к уравнениям в частных производных изучаются симметрии и законы сохранения уравнения Лиувилля. Интерес к уравнению Лиувилля и его свойствам в последнее время возрос в связи с его важной ролью в функциональной механике, предложенной И.В. Воловичем.

Symmetries and conservation laws of the Liouville equation are studied in the frames of the algebra-geometrical approach to partial differential equations. Interest in the Liouville equation and its properties has intensified recently due to its important role in functional mechanics proposed by I.V. Volovich.

## 16:00–16:30 (MSK), 14:00–14:30 (CET)

**A.K. Gushchin**
*Steklov Mathematical Institute, Russia*
**Title:** О задаче Дирихле
On the Dirichlet problem
**Abstract:** Основным содержанием настоящего доклада является расширение понятия решения задачи Дирихле: увеличения множества граничных функций и изменение определения принятия граничного условия, которые включают классическое и обобщенное решения. Для описания места обсуждаемых результатов среди известных придется привести ряд хорошо известных утверждений, обсудить основные постановки задачи Дирихле, их преимущества и недостатки. В центре внимания будут условия на данные задачи: правую часть уравнения, граничную функцию, коэффициенты уравнения и область, в которой задача рассматривается.

## 16:30–17:00 (MSK), 14:30–15:00 (CET)

**V.P. Burskii**
*Moscow Institute of Physics and Technology, Russia*
**Title:** Некоторые результаты из общей теории граничных задач для дифференциальных уравнений в частных производных
Some results from the general theory of boundary value problems for PDEs
**Abstract:** Mathematical modeling is now a universal hobby in the world. As is known, any advanced mathematical model is based on the differential equation (or a system of such equations), but in order to from a huge set of solutions select one that describes the future behavior of the modeled object, a boundary (initial) problem should be set. How to correctly set

the boundary problem, properties of boundary problems and differential equations - studies the theory boundary value problems, which is currently developing only in the direction study of three types of equations and a small number of formulations of boundary value problems. What happens in the general case, for more general equations and boundary problems, studies the general theory of boundary value problems, about which little is known to the public. And this theory is evolving. Here's about this theory and some new results author and there will be a story in my speech.

## 17:30–18:00 (MSK), 15:30–16:00 (CET)

**M.O. Katanaev**
*Steklov Mathematical Institute, Russia*
**Title:** О разделении переменных в уравнении Гамильтона-Якоби
On the separation of variables in the Hamilton-Jacobi equation
**Abstract:** We consider a point particle moving on a manifold equipped with a metric of arbitrary signature. Necessary and sufficient conditions on metric admitting complete separation of variables in the Hamilton-Jacobi equation are found in the case of linear and indecomposable quadratic conservation laws.

## 18:00–18:30 (MSK), 16:00–16:30 (CET)

**V.V. Vedenyapin**
*Keldysh Institute of Applied Mathematics, Russia*
**Title:** О выводе уравнений электродинамики и гравитации из принципа наименьшего действия, методе Гамильтона-Якоби и космологических решениях
On the derivation of the equations of electrodynamics and gravitation from the principle of least action, the Hamilton-Jacobi method and cosmological solutions

**Abstract:** TBA

# 10 January, Tuesday

## 11:00–11:30 (MSK), 09:00–09:30 (CET)

**D.V. Treschev**
*Steklov Mathematical Institute, Russia*
**Title:** Normalization flow
**Abstract:** I propose a new approach to the theory of normal forms for Hamiltonian ODE systems near a non-degenerate equilibrium position. The traditional normalization procedure is performed step-by-step: non-resonant terms in the expansion of the Hamiltonian function are removed first in the lowest degree, then in the next one and so on. I consider the space of all Hamiltonian functions with equilibrium position at the origin and construct a differential equation in this space. Solutions of this equation move Hamiltonian functions towards their normal forms. Shifts along the flow of this equation correspond to canonical coordinate changes. So, we have a continuous normalization procedure. The formal aspect of the theory presents no difficulties. The analytic aspect and the problems of convergence of series, as usual, non-trivial.

## 11:30–12:00 (MSK), 09:30–10:00 (CET)

**A.N. Pechen**
*Steklov Mathematical Institute and NITU MISIS, Russia*
**Title:** ENVIRONMENT AS A RESOURCE FOR CONTROLLING QUANTUM SYSTEMS
**Abstract:** In this talk, we consider environment as a resource for controlling quantum systems. We will discuss a speed-up

of generating mixed states for an open qubit using coherent control and time-dependent incoherent control found with GPM approach, as well as using gradient search for this problem. We will present rigorous results about convergence of the speed gradient method to optimal solution for a dissipative quantum oscillator driven by coherent and incoherent controls, and discuss conditions on the parameters of the method which guarantee that the obtained controls are physical. In addition, we will discuss recent results on Hessian structure of quantum control landscape for single qubit gate generation.

## 12:00–12:30 (MSK), 10:00–10:30 (CET)

**A.S. Trushechkin**
*Steklov Mathematical Institute, Russia*
**Title:** Квантовые динамические полугруппы для систем с квазивырожденными уровнями
Quantum dynamical semigroups for systems with quasidegenerate levels
**Abstract:** Одним из классических результатов в современной математической физике является строгий вывод Э. Дэвисом (E. B. Davies) в 1974 г. линейного кинетического уравнения для открытой квантовой системы, слабо взаимодействующей с резервуаром. Этот результат интересен как с фундаментальной точки зрения, так и с прикладной. Фундаментальный интерес связан со строгим обоснованием термодинамики и необратимой динамики. Прикладной интерес связан с активно развивающимися сейчас квантовыми технологиями. Квантовые системы, с которыми имеют дело квантовые технологии, не являются изолированными, а взаимодействуют с окружением. Однако теорема Дэвиса имеет некоторые предположения, которые часто не

выполняются на практике: например, предположение об отсутствии близких собственных значений в спектре гамильтониана квантовой системы. Множество работ было посвящено попыткам преодолеть это ограничение. В докладе будет рассказано об обобщении теоремы Дэвиса для случая, когда в спектре могут присутствовать близкие собственные значения.

## 13:00–13:30 (MSK), 11:00–11:30 (CET)

**Yu.N. Drozhzhinov**
*Steklov Mathematical Institute, Russia*
**Title:** Многомерная тауберова теорема В.С. Владимирова и ее развитие в МИАН
The multidimensional Tauberian theorem of V.S. Vladimirova and her development at MIAN
**Abstract:** TBA

## 13:30–14:00 (MSK), 11:30–12:00 (CET)

**Stevan Pilipovic**
*University of Novi Sad, Serbia*
**Title:** Contributions to the convolution and $\Psi$DO's over ultradistribution spaces
**Abstract:** The convolution of distributions was studied from the early beggining of the distribution theory, by many authors. Important contribution was given by Professor Vladimirov. I have studied, with my students, convolution in various spaces of distributions and ultradistributions. The aim of this talk is to show that one can extend the Anti-Wick calculus over $\mathcal D^{\{M_p\}}(\RR^d)$ for ultradistributions in

${\mathcal S'}_{\{A_p\}}^{\{M_p\}}(\RR^d)$ with very weak assumptions on $A_p$ and conditions on $M_p$ related to the sequence $p!^m, m>1$ noted in the abstract. This is done by the use of the Wigner transform $W(\varphi,\varphi)$ with $\varphi $ being ultradifferentiable functions with the fast decrease as $|x|\rightarrow \infty.$ We develop the theory for $\varphi=e^{-r{\langle \cdot\rangle^q}},\; r>0, q\geq 1,$ as well as for $\varphi$ satisfying even faster decay. Special example is $\varphi= \exp{(-se^{{\langle\cdot\rangle}^q})}, s>0,q\geq 1.$ Note that we have given earlier a complete answer in our analysis related to the convolution with the kernel $e^{a|\cdot|^q},a>0$ and the related Anti-Wick calculus, in the case when $\varphi$ is a Gaussian.

## 14:00–14:30 (MSK), 12:00–12:30 (CET)

**Xiangyu Zhou**
*Institute of Mathematics, Academy of Mathematics and Systems Science, China*
**Title:** Recent results in several complex variables and complex geometry
**Abstract:** We'll talk about some recent results in several complex variables and complex geometry, e.g., the solution of Demailly's strong openness conjecture on multiplier ideal sheaves, converse of L^2 existence theorem due to Hoermander and Demailly, and their applications in some fundamental problems in complex geometry.

## 15:30–16:00 (MSK), 13:30–14:00 (CET)

**Hiroshi Kaneko**
*Tokyo University of Science, Japan*
**Title:** Increase in Sobolev norm induced by composite with normal contraction on a ultrametric space

**Abstract:** It is well-known in the theory of Dirichlet form theory that every Dirichlet form and its domain provide us with a counterpart of $H^1$-space and the space possesses non-increasing property of $H^1$-norm under normal contraction, i.e., the $H^1$-norm of composite of normal contractions with function in the domain does not exceed the $H^1$-norm of the function without composite of normal contraction. Accordingly, it might be expected that so does such counterpart of the Sobolev space initiated by M. Fukushima and the speaker on the basis of transition semigroup kernels. In this talk, we find a counter-example to such a non-increasing property of the probabilistic Sobolev norm, i.e., the composite of normal contraction with some function in such Sobolev space induces increase in the norm.

## 16:00–16:30 (MSK), 14:00–14:30 (CET)

**A.A. Grigor'yan**
*Bielefeld University, Germany*
**Title:** Анализ на ультра-метрических и фрактальных пространствах
Analysis on ultra-metric and fractal spaces
**Abstract:** Это будет об оценках ядер теплопроводности на таких пространствах. В частности, есть такое интересное наблюдение, что регулярные метрические пространства можно охарактеризовать (по крайней мере с точки зрения уравнения теплопроводности) параметром, который называется walk dimension, и который принимает следующие значения: для Евклидовых пространств это 2, для ультра-метрических пространств (включая p-адические числа) это бесконечность, а весь промежуток от 2 до бесконечности занимают фракталы.

## 16:30–17:00 (MSK), 14:30–15:00 (CET)

**Alexander Bendikov**
*FAU Erlangen-Nuerenberg, Germany*
**Title:** Hierarchical Schrödinger operators with singular potentials
**Abstract:** This talk is based on the joint paper Hierarchical Schrödinger operators with singular potentials by Alexander Bendikov, Alexander Grigor'yan and Stanislav Molchanov. The goal of this paper is twofold. We prove that the operator $H=L+V$, the perturbation of the Taibleson-Vladimirov multiplier $L=D^{α}$ by the potential $V(x)=b‖x‖^{-α}, b≥b_{∗}$, is closable and its minimal closure is a non-negative definite self-adjoint operator (the critical value $b_{∗}$ depends on $α$ and will be specified in the paper). While the operator $H$ is non-negative definite the potential $V(x)$ may well take negative values, e.g.$ b_{∗}<0$ for all $0<α<1$. The equation $Hu=v$ admiits a Green function $g_{H}(x,y)$, the integral kernel of the operator $H^{-1}$. We obtain sharp lower- and upper bounds on the ratio of the functions $g_{H}(x,y)$ and $g_{L}(x,y)$. Examples illustrate our exposition.

## 17:30–18:00 (MSK), 15:30–16:00 (CET)

**L.M. Kozhevnikova**
*Sterlitamak Branch of Bashkir State University, Russia & Elabuga Branch of Kazan (Volga region) Federal University, Russia*
**Title:** О решениях нелинейных эллиптических уравнений с L_1-данными в неограниченных областях
On solutions of nonlinear elliptic equations with L_1-data in unbounded domains

**Abstract:** В докладе будут рассмотрены эллиптические уравнения второго порядка с различными видами нелинейностей и правой частью из пространства $L_1(Ω)$ в неограниченных областях $Ω$. Мера области Ω может быть как конечной, так и бесконечной. Для степенных нелинейностей (как постоянных, так и переменных) соответствующие пространства являются рефлексивными и вопросы существования и единственности энтропийных и ренормализованных решений хорошо изучены.

Нестепенные нелинейности определяются функциями Музилака-Орлича $M(x,s)$ и без дополнительных условий регулярности по второй переменной соответствующее пространство не обязано быть рефлексивным. Для эллиптических уравнений с такими нелинейностями известны результаты существования и единственности энтропийных и ренормализованных решений, а также их эквивалентности для областей с конечной мерой. В докладе будут приведены результаты, полученные автором по данному направлению исследований.

## 18:00–18:30 (MSK), 16:00–16:30 (CET)

**Yu.A. Alkhutov[a], G. A. Chechkin[b]**

[a] *A.G., N.G.Stoletov Vladimir State University, Russia*

[b] *M.V.Lomonosov Moscow State University, Russia*

**Title:** Elliptic Equations and Meyers Estimates

**Abstract:** This work is connected with estimates of solutions to the Zaremba problem for elliptic equation in bounded Lipschitz domain $D\in \mathbb{R}^n$, where $n>1$, of the form \begin{equation}\label{op} \mathcal{L}u:=\text{div}

(|\nabla u|^{p-2}a(x)\nabla u) \end{equation} with uniformly elliptic measurable and symmetric matrix $a(x)=\{a_{ij}(x)\}$, i.e. $a_{ij}=a_{ji}$ and \begin{equation}\label{1} \alpha^{-1}|\xi|^2\le \sum\limits_{i,j=1}^na_{ij}(x)\xi_i\xi_j\le\alpha |\xi|^2~\mbox{for almost all}~x\in D~\mbox{and all}~\xi\in \mathbb{R}^n. \end{equation} We assume that $F\subset\partial D$ is closed and $G=\partial D\setminus F$. Consider the Zaremba problem \begin{equation}\label{2} \left\{\begin {array}{l} \mathcal{L}u=l\quad \text{in}\quad D,\\ u=0\quad \text{on}\quad F,\\ \frac{\partial u}{\partial \nu}=0\quad \text{on}\quad G, \end{array}\right. \end{equation} where $\frac{\partial u}{\partial \nu}$ is the outer conormal derivative of $u$, and $l$ is a linear functional on $W^1_p(D, F)$, the completion of the set of infinitely differentiable in the closure of $D$ functions vanishing in the vicinity of $F$, by the norm $$ \parallel u\parallel_{W^{1}_p(D, F)}=\biggl (~\int\limits_{D} u^p\,dx+\int\limits_{D}|\nabla u|^p\,dx\biggr )^{1/p}. $$ By the solution of the problem \eqref{2} we mean the function $u \in W^1_p (D, F)$ for which the integral identity \begin{equation}\label{3} \int\limits_{D}|\nabla u|^{p-2}a\nabla u\cdot\nabla\varphi\,dx=\int\limits_{D} f\cdot\nabla\varphi\,dx \end{equation} holds for all test-functions $\varphi\in W^1_p(D, F)$, the components of the vector-function $f=(f_1,\ldots,f_n)$ belong to $L_{p'}(D)$, $p'=\frac{p}{p-1}$. For the compact $K\subset \mathbb{R}^n$ we define the capacity $C_q(K)$, $1<q<n$, by the formula \begin{equation}\label{hu} C_q(K)=\inf~ \biggl \{~ \int\limits_{\mathbb{R}^n}|\nabla\varphi|^q\,dx:~\varphi\in C^\infty_0 (\mathbb{R}^n),~\varphi\ge 1~\mbox{on}~K\biggr \}, \end{equation} if $p\in (1, n/(n-1)]$, then $q=(p+1)/2$, but if $r\in (n/(n-1), n]$, where $n > 2$, then $q=np/(n+p)$. $\bullet$ {\bf Case of linear equation ($p=2$).} Suppose $B^{x_0}_r$ is an open ball of the radius $r$ centered in $x_0$, and $mes_{n-1}(E)$ is $(n-1)$-measure of the set $E$. Assume

also that $q=2n/(n+2)$ as $n>2$ and $q=3/2$ as $n=2$. We suppose one of the following conditions is fulfilled: for an arbitrary point $x_0\in F$ as $r\le r_0$ the inequality \begin{equation}\label{g1} C_q( F\cap \overline B^{x_0}_r)\ge c_0 r^{n-q} \end{equation} holds true or the inequality \begin{equation}\label{g2} mes_{n -1}( F\cap \overline B^{x_0}_r)\ge c_0 r^{n-1} \end{equation} holds, the positive constant $c_0$ does not depend on $x_0$ and $r$. Condition \eqref{g2} is universal (even for nonlinear equations). \begin{theo} If $f\in L_{2+\delta_0}(D)$, where $\delta_0>0$, then there exist positive constants $\delta(n,\delta_0)<\delta_0$ and $C$, such that for a solution to the problem \eqref{2} the estimate \begin{equation}\label{t} \int\limits_{D}|\nabla u|^{2+\delta}dx\leq C\int\limits_{D}|f|^{2+\delta}\ dx, \end{equation} holds, where $C$ depends only on $\delta_0$, the dimension $n$, constant $c_0$ from \eqref{g1} and \eqref{g2}, and also the constant $r_0$. \end{theo} $\bullet$ {\bf Case of $p$-elliptic equation ($p>1$).} \textbf{A.} If $1< p \le n$, then the following condition is assumed to hold: for an arbitrary point $x_0\in F$ for $r\le r_0$, the condition \eqref{g1} is true. \textbf{B. }If $p> n$, then the set $F$ is assumed to be nonempty: $F\neq \emptyset$. \begin{theo} If $f\in L_{p'+\delta_0}(\Omega)$, where $\delta_0>0$, then there exist positive constants $\delta(n, p,\delta_0)<\delta_0$ and $C$, such that for a solution to the problem \eqref{2} the estimate \begin{equation}\label{tm} \int\limits_{\Omega}|\nabla u|^{p+\delta}dx\leq C\int\limits_{\Omega}|f|^{p'(1+\delta/p)}\ dx, \end{equation} holds, where $C$ depends only on $p$, $\delta_0$, the dimension $n$, constant $c_0$ from \eqref{g1} and \eqref{g2}, and also the constant $r_0$. \end{theo}
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\begin{center} {\bf Bibliography.} \end{center}
{[1] Yu.A. Alkhutov, G.A. Chechkin, The Meyer's Estimate of Solutions to Zaremba Problem for Second-order Elliptic Equations in

Divergent Form~//  C\,R M\'ecanique, T. 349 (2, 2021). P. 299--304.} {[2] Yu.A. Alkhutov, G.A. Chechkin, V.G. Maz'ya. On the Boyarsky--Meyers Estimate of a Solution to the Zaremba Problem // Arch Rational Mech Anal, V. 245, No 2 (2022). P. 1197--1211.} {[3] Yu.A. Alkhutov, A.G. Chechkina. Many-Dimensional Zaremba Problem for an Inhomogeneous $p$-Laplace Equation // Russian Academy of  Sciences. Doklady Mathematics, V. 106, No 1 (2022). P. 143--146.}

# 11 January, Wednesday

## 11:00–11:30 (MSK), 09:00–09:30 (CET)

**V.V. Kozlov**
*Steklov Mathematical Institute, Russia*
**Title:** Дискретные симметрии уравнений динамики с полиномиальными интегралами высших степеней
Discrete symmetries of equations of dynamics with polynomial integrals of higher degrees
**Abstract:** Рассматриваются системы с торическим конфигурационным пространством и кинетической энергией в виде “плоской” римановой метрики на торе. Потенциальная энергия $V$ – гладкая функция на конфигурационном торе. Динамика таких систем описывается “натуральными” гамильтоновыми системами дифференциальных уравнений. Если заменить $V$ на $\epsilon V$,где $\epsilon$– малый параметр, то исследование таких гамильтоновых систем при малых значениях $\epsilon$ относится к “основной проблеме динамики” по Пуанкаре. Обсуждается известная гипотеза об однозначных полиномиальных по импульсам

интегралах уравнений движения: если имеется полиномиальный по импульсам интеграл степени $m$, то обязательно найдется линейный или квадратичный по импульсам первый интеграл. Она полностью доказана для $m= 3$ и $m= 4$. Обсуждаются случаи “высших” степеней, когда$ m= 5$ и $m= 6$. Следуя теории возмущений гамильтоновых систем, вводятся резонансные прямые на плоскости импульсов. Если система допускает полиномиальный интеграл, то число этих прямых конечно. Найдены симметрии множества резонансных прямых, что даёт, в частности, необходимые условия интегрируемости. Получены некоторые новые критерии существования однозначных полиномиальных интегралов.

## 11:30–12:00 (MSK), 09:30–10:00 (CET)

**A.I. Aptekarev**
*Keldysh Institute of Applied Mathematics, Russia*
**Title:** Возникновение иерархии особенностей в средах с постоянным давлением и крупномасштабная структура Вселенной
The Emergence of a Hierarchy of Singularities in Media with Constant Pressure and the Large-Scale Structure of the Universe
**Abstract:** TBA

## 12:00–12:30 (MSK), 10:00–10:30 (CET)

**A.G. Sergeev**
*Steklov Mathematical Institute, Russia*

**Title:** Topological phases in the theory of solid states
**Abstract:** This paper is devoted to the theory of topological phases --- a new and actively developing direction in solid state physics. The topological phases are defined in the following way. Denote by $G$ the symmetry group and consider the set $\text{Ham}_G$ of classes of homotopy equivalent $G$-symmetric Hamiltonians. We assume that they have the energy gap stable under small deformations which makes it reasonable to use the topological methods for their study. It is possible to introduce on $\text{Ham}_G$ a natural stacking operation such that $\text{Ham}_G$, provided with this operation, becomes an Abelian monoid (i.e. an Abelian semigroup with the neutral element). The group of invertible elements of this monoid is precisely the topological phase. The initial ideas, lying in the base of the theory of topological phases, were formulated by Alexei Kitaev in his talks. It turns out that the family $(F_d)$ of $d$-dimensional topological phases forms an $\Omega$-spectrum. In other words, it has the property that the loop space $\Omega F_{d+1}$ is homotopy equivalent to the space $F_d$. This fact opens a way to wide use of algebraic topology methods for the study of topological phases. More concretely, one can associate with any $\Omega$-spectrum the generalized cohomology theory, determined by the functor $h^d$, which assigns to the topological space $X$ the set $[X,F_d]$ of classes of homotopy equivalent maps $X\to F_d$.

## 13:00–13:30 (MSK), 11:00–11:30 (CET)

**I.Ya. Aref'eva**
*Steklov Mathematical Institute, Russia*
**Title:** Non-Local Equations in dS-spacetime
**Abstract:** TBA

## 13:30–14:00 (MSK), 11:30–12:00 (CET)

**A.S. Koshelev**
*Universidade da Beira Interior, Portugal*
**Title:** Stability of scalar fields with complex masses in curved backgrounds
**Abstract:** In my talk I will consider scalar fields with complex masses in curved backgrounds. Such fields arise in field theories with higher (and especially infinite) derivatives and their presence is currently under an active study. In my talk I will show certain constraints on the background metrics when these strange new fields do not feature growing modes. A connection with quantization and unitarity of the models will be discussed as well.

## 14:00–14:30 (MSK), 12:00–12:30 (CET)

**Kh. A. Khachatryan**
*Yerevan State University, Armenia*
**Title:** О нелинейных интегральных уравнениях в теории p-адических струн
On nonlinear integral equations in the theory of p-adic strings
**Abstract:** Доклад посвящен вопросам единственности и существования, а также отсутствия нетривиального ограниченного решения для двух классов нелинейных интегральных уравнений в теории p-адических струн. Мы будем исследовать также некоторые качественные свойства построенного решения: монотонность, непрерывность, выпуклость, гладкость и асимптотическое поведение на бесконечности. В конце доклада приведем конкретные частные примеры указанных уравнений.

## 15:30–16:00 (MSK), 13:30–14:00 (CET)

**Richard Kerner**
*University of Paris, France*
**Title:** Ternary Z3-graded Generalization of Heisenberg's Algebra
**Abstract:** Many Z2-graded algebraic structures can be generalized to the case of Z3-grading. Several such structures are presented: the Grassmann algenra, algebra of exterior forms, lie algebras. In the latter, the antisymmetric binary product is replaced by ternary Z3-skew symmetric product. Ternary Heisenberg algebra is then introduced and its Bogolyubov symmetry group established. The second quantization of this structure is introduces, and the sixth-order Hamiltonian defined, along with its eigenstates. The Bohr-Sommerfeld quantization is applied to the stationary periodic solutions and the eigenvalues of energy are computed.

## 16:00–16:30 (MSK), 14:00–14:30 (CET)

**T.V. Dudnikova**
*Keldysh Institute of Applied Mathematics, Russia*
**Title:** О стационарных неравновесных состояниях линейных гамильтоновых систем
On stationary nonequilibrium states of linear Hamiltonian systems
**Abstract:** In the talk, we discuss the long-time behavior of distributions of solutions for infinite-dimensional Hamiltonian systems and the existence of a nonzero heat flux in them. As a model, we consider a linear Hamiltonian system consisting of a real scalar Klein-Gordon field coupled to an infinite harmonic crystal. This system can be considered as the

description of the motion of electrons (so-called Bloch electrons) in the periodic medium that is generated by the ionic cores. For the coupled system, we study the Cauchy problem with random initial data. We prove that the distributions of the solutions weakly converge to a limiting measure for large times. Under the condition that the initial random function in the ``left'' and ``right'' parts of the space has the Gibbs distribution with different temperatures, we find the stationary states (i.e., the probability limiting measures) of the system in which the limiting energy current density does not vanish. Thus, for this system we construct a class of stationary non-equilibrium states.

## 16:30–17:00 (MSK), 14:30–15:00 (CET)

**N.G. Marchuk**
*Steklov Mathematical Institute, Russia*
**Title:** Some relativistic invariant systems of differential equations
**Abstract:** We define a new class of partial differential equations of first order (complex covariantly equipped systems of equations), which are invariant with respect to (pseudo)orthogonal changes of Cartesian coordinates of (pseudo)euclidian space. It is shown that for pseudoeuclidian spaces of signature (1,n-1) covariantly equipped systems of equation can be written in the form of Friedrichs symmetric hyperbolic systems of equations of first order. We prove that Maxwell and Dirac model equations belong to the class of covariantly equipped systems of equations.

## 17:00–17:30 (MSK), 15:00–15:30 (CET)

**A.Yu. Savin**
*Peoples' Friendship University of Russia, Russia*

**Title:** Некоммутативные вычеты и асимптотики следов операторов, ассоциированных с метаплектической группой
Noncommutative residues and asymptotics of traces of operators associated with a metaplectic group
**Abstract:** We consider the operator algebra generated by pseudodifferential operators in Rn and quantizations of isometric affine canonical transformations. For elements in this algebra, we study regularized traces defined in terms of zeta-regularizations. We construct analogues of the well-known Wodzicki residue on this operator algebra. The results were obtained in a joint work with Elmar Schrohe (Hannover). The work was partially supported by RFBR, project Nr. 21-51-12006.

# 12 January, Thursday

## 11:00–11:30 (MSK), 09:00–09:30 (CET)

**V.P. Maslov[a], S.Yu. Dobrokhotov[b], V.E. Nazaikinskii[b], A.I. Shafarevich[a,c]**

[a] *Moscow State University, Russia*

[b] *Ishlinsky Institute for Problems in Mechanics, Russia*

[c] *Steklov Mathematical Institute, Russia*

**Title:** Современное состояние теории канонического оператора
The current status of the theory of the canonical operator
**Abstract:** Канонический оператор, разработанный в 1965 г. одним из докладчиков, является одним из наиболее мощных средств построения глобальных квазиклассических асимптотик. В докладе будет рассказано о недавних продвижениях в конструкции канонического оператора, позволяющих, в сочетании с

системами технических вычислений, такими как Wolfram Mathematica, применять его как эффективный вычислительный инструмент в конкретных задачах.

## 11:30–12:00 (MSK), 09:30–10:00 (CET)

**I.A. Taimanov**
*Sobolev Institute of Mathematics, Russia & Novosibirsk State University, Russia*
**Title:** Quasiclassical approximation for magnetic monopoles
**Abstract:** A quasiclassical approximation is constructed to describe the eigenvalues of the magnetic Laplacian on a compact Riemannian manifold in the case when the magnetic field is not given by an exact 2-form. For this, the multidimensional WKB method in the form of Maslov canonical operator is applied. In this case, the canonical operator takes values in sections of a nontrivial line bundle.

## 12:00–12:30 (MSK), 10:00–10:30 (CET)

**S.Yu. Dobrokhotov**
*Ishlinsky Institute for Problems in Mechanics, Russia*
**Title:** Конструктивные асимптотики для линейных (псевдо)дифференциальных уравнений с локализованными правыми частями
Constructive asymptotics for linear (pseudo)differential equations with localized right-hand sides
**Abstract:** Обсуждается метод построения квазиклассических асимптотических решений неоднородных дифференциальных и псевдодифференциальных уравнений (с частными производными) с локализованными правыми частями. Эти задачи близки к задачам об асимптотике функции Грина для таких операторов, в частности, изученным в

многочисленных работах задачах об асимптотике функции Грина для уравнения Гельмгольца. Метод основан на конструктивном описании С.Ю.построения эффективных асимптотических решений указанных задач, возникающих в различных областях физики и механики сплошных сред. Метод иллюстрируется примерами для уравнения Шредингера и псевдодифференциального уравнения теории волн на воде. Обсуждаемые результаты получены совместно с А.Ю.Аникиным, В.Е.Назайкинским, М.Руло и А.А.Толченниковым в рамках госзадания № АААА-А20-120011690131-7 и частичной поддержке Российским научным фондом (проект 21-11-00341).

## 13:00–13:30 (MSK), 11:00–11:30 (CET)

**A.I. Shafarevich**
*Moscow State University, Russia & Steklov Mathematical Institute, Russia*
**Title:** Lagrangian manifolds and complex vector bundles, corresponding to semi-classical solutions for equations with delta-type singularities
**Abstract:** Semi-classical asymptotics of solutions for a wide class of evolution equations with smooth coefficients are connected with geometric objects — Lagrangian surfaces or complex vector bundles over isotropic manifolds. If the coefficients of the equations contain singularities (or depend singularly on a small parameter of the problem), geometric objects are rebuilt on the sets corresponding to the supports of these singularities. The talk discusses the form of asymptotic solutions and rearrangements of geometric objects for certain examples of evolutionary problems with singularities.

## 13:30–14:00 (MSK), 11:30–12:00 (CET)

**V.E. Nazaikinskii**
*Ishlinsky Institute for Problems in Mechanics, Russia*
**Title:** Uniformization of degenerate equations and semiclassical asymptotics
**Abstract:** For a stratified manifold $X$ that is the quotient of a smooth compact manifold $M$ by a compact Lie group action, we define smooth functions on $X$ as invariant smooth functions on $M$ and $h$-(pseudo)differential operators on $X$ as the restrictions of invariant $h$-(pseudo)differential operators on $M$ to the space of smooth functions on $X$. We discuss semiclassical asymptotics for pseudodifferential equations on $X$; examples include boundary degenerate linearized shallow water equations describing the run-up of long waves on a shallow beach and $h$-pseudodifferential equations on orbifolds.

## 14:00–14:30 (MSK), 12:00–12:30 (CET)

**Yu. A. Kordyukov**
*Institute of Mathematics, Ufa Federal Research Centre, Russia*
**Title:** Semiclassical asymptotics of the spectral function of the magnetic Schrodinger operator
**Abstract:** In the talk, we discuss asymptotic spectral properties of the Schrödinger operator with uniformly bounded magnetic field in Euclidean space in the semiclassical limit. We give a rough asymptotic description of its spectrum and describe the full off-diagonal asymptotic expansion of its smoothed spectral function. As consequences, we obtain the semiclassical trace formula and an asymptotic localization property of the spectral function in the case when the magnetic field has maximal rank.

# 13 January, Friday

## 11:30–12:00 (MSK), 09:30–10:00 (CET)

**N.B. Engibaryan**
*Institute of Mathematics of National Academy of Sciences of Armenia, Armenia*
**Title:** О сочетании интегралов Лебега и Римана в теории уравнений свертки
On the combination of Lebesgue and Riemann integrals in the theory of convolution equations
**Abstract:** Математическая теория интегральных уравнений свертки в основном построена на базе интеграла Лебега (ИЛ). Вопросы разрешимости уравнений свертки, в которых фигурирует собственный или несобственный интеграл Римана, часто приходится рассматреть в лебеговых функциональных пространствах. С другой стороны, прикладные возможности интеграла Лебега существенно уступают интегралу Римана (ИР). В докладе представлены способы эффективного сочетания теоретических возможностей ИЛ и прикладных возможностей ИР в вопросе решения некоторых известных уравнений свертки. Ориентиром при рассмотрении данного круга вопросов послужили работы В.А.Амбарцумяна и В.С. Владимирова по уравнениям переноса; проблема актуальной бесконечности. Применяются: нелинейные уравнения факторизации автора, метод усреднения ядра автора с А.Г.Барсегян и др.

## 12:00–12:30 (MSK), 10:00–10:30 (CET)

**A.L. Skubachevskii**
*Peoples' Friendship University of Russia, Russia*
**Title:** Априорные оценки решений системы уравнений Власова-Пуассона для двухкомпонентной плазмы
A priori estimates for solutions to a system of equations Vlasov-Poisson for two-component plasma
**Abstract:** Рассмотрена смешанная задача для системы уравнений Власова--Пуассона, описывающая кинетику высокотемпературной плазмы в термоядерном реакторе при воздействии внешнего магнитного поля. Получена априорная оценка для решения данной смешанной задачи с компактными по пространственным переменным носителями функций плотности распределения заряженных частиц.

## 13:00–13:30 (MSK), 11:00–11:30 (CET)

**V.A. Zagrebnov**
*Institute of Mathematics of Marseille, France*
**Title:** Замечание об асимптотике Гиббсовских полугрупп в нуле
A note about asymptotics of Gibbs semigroups at the origin
**Abstract:** Установлена связь между асимптотикой Гиббсовских полугрупп в нуле и свойствами резольвенты генераторов этих полугрупп. Основано на совместной работе с Брюно Ёкумом (Центр теоретической физики, Марсель)

We established a connection between asymptotic behaviour of the Gibbs semigroups at the origin and the properties of resolvent of their generators. Based on the project with Bruno Iochum (Centre de Physique Théorique, Marseille)

## 13:30–14:00 (MSK), 11:30–12:00 (CET)

**V.Zh. Sakbaev**
*Keldysh Institute of Applied Mathematics, Russia*
**Title:** Аналог теоремы Якоби для бесконечномерных торов
Analogue of Jacobi's theorem for infinite-dimensional tori
**Abstract:** Изучаются динамические свойства бесконечной системы гармонических осцилляторов. Как и в классической теореме Якоби о траекториях конечной системы осцилляторов на торе, получены условия периодичности, неблуждаемости и транзитивности на инвариантном торе траекторий бесконечной системы осцилляторов. Изучается эргодичность меры и эргодичность пространства функций на инвариантном многообразии относительно потока системы осцилляторов. Доклад основан на совместной работе с И.В. Воловичем.

Dynamic properties of an infinite system of harmonic oscillators are studied. As in the classical Jacobi theorem on the trajectories of a finite system of oscillators on a torus, the conditions of periodicity, non-wandering and transitivity on an invariant torus of trajectories of an infinite system of oscillators are obtained. The ergodicity of the measure and the ergodicity of the space of functions on an invariant manifold with respect to the flow of a system of oscillators are studied. The talk is based on joint work with I.V. Volovich.

## 14:00–14:30 (MSK), 12:00–12:30 (CET)

**Paolo Gibilisco**
*University of Rome Tor Vergata, Italy*
**Title:** Means in quantum physics and somewhere else

**Abstract:** Chentsov created Information Geometry by showing that classical Fisher Information is the only metric on the spaces of probability vectors which ``contracts under noise". Chentsov himself posed the same question about the quantum realm and was Petz to find the answer using means. Still using means Petz clarified the notion of quantum variance. I'll discuss how means enable us to understand the relation between Quantum Variance and Quantum Fisher Information and how this sheds new light on the Uncertainty Principle. \cite{G22b}
{\bf The Jensen inequality for means} Recently the Jensen inequality for 2-means has been proved. Does a general version for $n$-means exist? \cite{G22a}
{\bf Stam inequality as a mean inequality} The Stam inequality for the Gamma distribution appears as a mean inequality. Can this be generalized?\cite{G14}
\begin{thebibliography}{99} \bibitem{G22b} \textsc{P. Gibilisco.} Uncertainty and Quantum Variance at the light of Quantum Information Geometry. {\it Information Geometry}, doi.org/10.1007/s41884-022-00087-5, 2022. \bibitem{G22a} \textsc{P. Gibilisco.} About the Jensen inequality for numerical $n$--means. {\it International Journal of Modern Physics A}, 2243010, 2022. \bibitem{G14} \textsc{P. Gibilisco}. Fisher information and means: some questions in the classical and quantum settings. {\it International Journal of Software and Informatics}, 8(3-4) pp. 265--276, 2014. \end{thebibliography}

## 15:30–16:00 (MSK), 13:30–14:00 (CET)

**S.V. Kozyrev**
*Steklov Mathematical Institute, Russia*
**Title:** Learning by population genetics and matrix Riccati equation
**Abstract:** A model of learning as generalization of the Eigen's quasispecies model in population genetics is introduced.

Eigen's model is considered as a matrix Riccati equation, the error catastrophe in the Eigen's model (when the purifying selection becomes ineffective) is discussed as the divergence of Perron--Frobenius eigenvalue of the Riccati model in the limit of large matrices. Known estimate for Perron--Frobenius eigenvalue provides explanation for observed patterns of genomic evolution. We propose to consider the error catastrophe in Eigen's model as analogue of overfitting in learning theory, this gives a criterion for presence of overfitting in learning.

## **16:00–17:00 (MSK), 14:00–15:00 (CET)**

# Memorial session