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Novosibirsk State University *THE REAL SCIENCE

PARISIAN RUIN WITH RANDOM DEFICIT-DEPENDENT DELAYS FOR SPECTRALLY NEGATIVE LÉVY PROCESSES

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We consider an interesting natural extension to the Parisian ruin problem under the assumption that the risk reserve dynamics are given by a spectrally negative Lévy process. The distinctive feature of this extension is that the distribution of the random implementation delay windows' lengths can depend on the deficit at the epochs when the risk reserve process turns negative, starting a new negative excursion. This includes the possibility of an immediate ruin when the deficit hits a certain subset. In this general setting, we derive a closed-from expression for the Parisian ruin probability and the joint Laplace transform of the Parisian ruin time and the deficit at ruin. This is joint work with Duy Phat Nguyen.

HARRIS ERGODICITY OF A SPLIT TRANSMISSION CONTROL PROTOCOL

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Additive-increase multiplicative-decrease transmission control protocols are well known and have been studied in numerous papers. It is much more difficult to study the properties of systems of interacting protocols. We consider a queueing system in which both the intensity of the input stream and the intensity of the service follow a TCP protocol and the dynamics of the latter depends on both intensities. This kind of stochastic system was proposed by Baccelli, Carofiglio, and Foss in 2009, who have proved the positive recurrence of the underlying Markov chain and studied a number of statistical properties of the model. In this paper, we introduce a more general stochastic model and prove a stronger statement: the Harris ergodicity of the corresponding Markov chain. This is joint work with Sergey Foss.

LOCAL PROBABILITIES FOR ASYMPTOTICALLY STABLE RANDOM WALKS IN HALF SPACE

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We consider an asymptotically stable multidimensional random walk $S(n) = (S_1(n), \ldots, S_d(n))$. Let $\tau_x := \min\{n > 0 : x_1 + S_1(n) \le 0\}$ be the first time the random walk $x + S(n)$ leaves the upper half-space. We study the asymptotics of $p_n(x, y) := \mathbb{P}(x + S(n) \in y + \Delta, \tau_x > n)$ as n tends to infinity, where Δ is a fixed cube. We obtain exact asymptotics in the regime of normal and small deviations and obtain accurate bounds in the regime of large deviations. From that we obtain the local asymptotics for the Green function of $G(x, y) := \sum_n p_n(x, y)$, as $|y|$ and/or $|x|$ tend to infinity. This is joint work with V. Wachtel.

LARGE DEVIATIONS FOR ASYMPTOTICALLY SPACE HOMOGENEOUS MARKOV CHAINS IN TWO DIMENSIONS

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We discuss Markov chains in the positive lattice quadrant whose transition probabilities converge at infinity. Assuming positive recurrence of the chain we study large deviations for its invariant probabilities under Cramer type conditions on jumps.

JOINT ASYMPTOTICS OF FORWARD AND BACKWARD PROCESSES OF NUMBERS OF NON-EMPTY URNS IN INFINITE URN SCHEMES

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We study the joint asymptotics of forward and backward processes of the numbers of non-empty urns in an infinite urn scheme. The probabilities of balls hitting the urns are assumed to satisfy the conditions of regular decrease. We prove weak convergence to a two-dimensional Gaussian process. Its covariance function depends only on the exponent of regular decrease of probabilities. The corollary of the main theorem asserts the weak convergence of the integral of the difference of forward and backward processes to the normal distribution. We obtain parameter estimates that have a joint normal distribution together with forward and backward processes.

We use these estimates to construct statistical tests for the homogeneity of the urn scheme on the number of thrown balls. We analyse the statistical tests by simulation and apply them to the analysis of the homogeneity of texts in natural language.

MODERATE DEVIATIONS PRINCIPLES FOR TRAJECTORIES OF INHOMOGENEOUS RANDOM WALKS

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We consider a normalized piecewise linear curve constructed from sums of independent random variables that may have different distributions. Under various moment conditions on random variables we present theorems containing the principles of moderate large deviations for such piecewise linear curves in the space of continuous functions on the interval [0,1]. We also point out the connection between the zone in which the principle of moderately large deviations is fulfilled and the moment that exists for random variables.

MULTI-NORMEX APPROACH FOR EVALUATING THE SUM OF HEAVY TAILED RANDOM VECTORS

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We build a sharp approximation of the whole distribution of the sum of iid heavy-tailed random vectors, combining mean and extreme behaviors. It extends the so-called 'Normex' approach from a univariate to a multivariate framework. We propose two possible multi-normex distributions, named d-Normex and MRV-Normex. Both rely on the Gaussian distribution for describing the mean behavior, via the CLT, while the difference between the two versions comes from using the exact distribution or the EV theorem for the maximum. The main theorems provide the rate of convergence for each version of the multi-normex distributions towards the distribution of the sum, assuming second order regular variation property for the norm of the parent random vector when considering the MRV-normex case. Numerical illustrations and comparisons are proposed with various dependence structures on the parent random vector, using QQ-plots based on geometrical quantiles. This is joint work with Marie Kratz.

DYNAMIC SYSTEMS RELATED TO THE EMERGENCE OF ALPHA-RHYTHM OF BRAIN CORTEX

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The following class of dynamic systems is studied: N points are rotating clockwise with speed equal to 1 on a unit circle. The connected oriented graph F with N nodes is given. The (unnown) real function on the circle $f(x)$ is given.There is a picked point 0 on the circle where $f(0) = 0$. The rotating points are making jumps as well: at moment t when any rotated point $n = 1, ..., N$ reaches 0, then each point m neighbouring by graph F to point n, jumps to the distance of $f(m(t))$ on the circle.The function $f(x)$ depends on N and the graph F is random. It is clear that these dynamic systems have a trivial invariant state when all the N points merge in one big atom rotating on the circle (without any jumps as $f(0) = 0$). Usually nontrivial states exist also for such dynamic systems. The problem is to find such a natural function $f(x)$ for which we shall converge to this trivial invariant state with high probability when t tends to infinity for growing N.

ON ASYMPTOTICS OF THE PROBABILITY FOR A RANDOM PROCESS TO STAY ABOVE A MOVING BOUNDARY

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Let $X_1, X_2, \ldots$ be independent random variables. We always assume that the random walk $S_n := X_1 + \ldots + X_n$, $n = 1, 2, \ldots$, belongs to the domain of attraction of the normal distribution: i.e. there exists an increasing to infinity sequence $\{b_n\}$ such that S_n/b_n converges in distribution towards the standard normal law as $n \to \infty$.

Let $T := \inf\{k \geq 1 : S_k \leq g_k\}$ be the first crossing time over the moving boundary $\{g_n = o(b_n)\}$ by the random walk $\{S_n\}$. We consider in the talk the asymptotic behavior of the upper tail $\mathbf{P}(T > n)$.

The known classical case is when random walks have zero means, finite variances and $B_n^2 := \mathbf{E}[S_n^2] \to \infty$. If the Lindeberg condition is satisfied then

$$\mathbf{P}(T > n) \sim \sqrt{\frac{2}{\pi}} \frac{U_n}{B_n} \quad \text{with} \quad U_n := \mathbf{E}[S_n - g_n; T_g > n]. \tag{1}$$

(See *Ann. Probab.*, 2018, pp. 3313-3350.)

In the present talk we focus on the further results in this direction.

In particular, we are not going to assume that all summands have finite variances or even finite expectations. Denote by $X_n^{[u_n]}$ the truncation of the random variable X_n on the levels $\pm u_n$, where $u_n/b_n \to 0$ sufficiently slow. In this case

$$\mathbf{P}(T > n) \sim \sqrt{\frac{2}{\pi}} \frac{U_n(u_n)}{b_n} + J_n(u_n, b_n), \tag{2}$$

where $U_n(u_n)$ is defined similar to U_n in (1), but for the random walk $X_1^{[u_n]} + \cdots + X_n^{[u_n]}$ instead of S_n. Note that the value $J_n(u_n, b_n)$ from (2) is found in explicit way as a function of distributions of positive jumps of random variables $X_1 - u_n, \ldots, X_n - u_n$.

The talk is based on the joint works with D. Denisov and V. Wachtel. The research was funded by RFBR and DFG according to the research project №20-51-12007.

HELLINGER INFORMATION MATRIX IN PARAMETRIC ESTIMATION AND OBJECTIVE PRIORS

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Hellinger information as a local characteristic of parametric distribution families was first introduced in (Shemyakin, 1992). It is related to the definition of Hellinger distance between two parametric values. Under certain regularity conditions, local behavior of the Hellinger distance is closely related to Fisher information and the geometry of Riemann manifolds. Nonregular distributions (non-differentiable distribution densities or undefined Fisher information), including uniform, require using analogues or extensions of Fisher information. Hellinger information may serve to construct information inequalities of Cramer-Rao type, extending the lower bounds of the Bayes risk (Borovkov and Sakhanenko, 1980) to the nonregular case (Shemyakin, 1991).

A construction of objective or non-informative priors based on Hellinger information was suggested in Shemyakin (2014). Hellinger priors extend the Jeffreys' rule to nonregular cases. For many examples, they are identical or close to the reference priors (Berger, Bernardo and Sun, 2009) or probability matching priors (Ghosal and Samanta, 1997). Most of the paper was dedicated to one-dimensional case, but the matrix definition of Hellinger information was also introduced for higher dimensions. Conditions of existence and nonnegative definite property of Hellinger information matrix were not discussed.

Hellinger information was also applied by Lin, Martin, and Yang (2019) to problems of optimal experimental design. A special class of parametric problems was considered, requiring directional definition of Hellinger information, but not a full construction of Hellinger information matrix. In the present paper, a general definition, existence and nonnegative definite property of Hellinger information matrix is considered for nonregular settings described in Ibragimov and Has'minskii (1981).

ON A KERNELS OF SOME RANDOM OPERATORS

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Let $\xi_x(t)$ be a solution of the stochastic differential equation

$$d\xi_x(t) = b(\xi_x(t))b'(\xi_x(t))\,dt + b(\xi_x(t))\,dw(t), \qquad \xi_x(0) = x.$$

In the space $L_2(\mathbb{R})$, consider the self-adjoint operator

$$\mathcal{A} = -\frac{1}{2}\frac{d}{dx}\big(b^2(x)\frac{d}{dx}\big) + V(x),$$

defined on the domain $W_2^2(\mathbb{R})$. Regarding the functions $b(x), V(x)$, we will assume that the following conditions are satisfied: 1. $V \in L_1(\mathbb{R})$. 2. $b \in C_b^2$ and separated from zero. 3. There exists $b_0 > 0$ such that $\lim\limits_{x\to\pm\infty} b(x) = b_0$. 4. $\lim\limits_{x\to\pm\infty} b'(x) = \lim\limits_{x\to\pm\infty} b''(x) = 0$. 5. $\int_{\mathbb{R}} x^2(|b(x) - b_0| + |b'(x)|)\,dx < \infty$.

The conditions 1-5 imply that the spectrum of the operator $\mathcal{A}$ consists of the interval $[0,\infty)$ and, possibly, several negative single eigenvalues. Denote by $H_a \subset L_2(\mathbb{R})$ an absolutely continuous subspace of the operator $\mathcal{A}$, and by P_a the orthogonal projection into $L_2(\mathbb{R})$ on H_a. Let $\mathcal{A}_0 = \mathcal{A}P_a$ denote the restriction of the operator $\mathcal{A}$ on H_a.

For each λ satisfying the condition $\operatorname{Re}\lambda \leqslant 0$, we define a random operator $\mathcal{R}_\lambda^t$ by setting

$$\mathcal{R}_\lambda^t f(x) = \int_0^t e^{\lambda\tau}(P_a f)(\xi_x(\tau))e^{-\int_0^\tau V(\xi_x(s))\,ds}\,d\tau.$$

Theorem 1. 1. With probability 1 the operator $\mathcal{R}_\lambda^t$ is a bounded integral operator in $L_2(\mathbb{R})$ of the form

$$\mathcal{R}_\lambda^t f(x) = \int_{\mathbb{R}} r_\lambda(t,x,y)f(y)\,dy,$$

where the last equality is also valid for $t = \infty$ in the case $\operatorname{Re}\lambda < 0$. 2. For any λ, t, x the function $r_\lambda(t,x,\cdot) \in W_2^\alpha$ for every $\alpha \in [0,\frac{1}{2})$.

Theorem 2. 1. If $\operatorname{Re}\lambda < 0$ then for all $f \in H_a$

$$\mathbb{E}\int_{\mathbb{R}} r_\lambda(\infty,\cdot,y)f(y)\,dy = (\mathcal{A}_0 - \lambda I)^{-1}f. \tag{1}$$

2. If $\operatorname{Re}\lambda = 0$ and $\lambda \neq 0$ then for all $f \in H_a$

$$\lim_{t\to\infty}\mathbb{E}\int_{\mathbb{R}} r_\lambda(t,\cdot,y)f(y)\,dy = (\mathcal{A}_0 - \lambda I)^{-1}f. \tag{2}$$

For $\lambda = 0$ the last equality (2) holds for every $f \in \mathcal{D}(\mathcal{A}_0 - \lambda I)^{-1}$.

The work is supported by RSF, project №22-21-00016.

INEQUALITIES FOR THE CHARACTERISTICS OF THE CUSUM PROCEDURE IN A CHANGE POINT PROBLEM

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We obtain an upper bound for the average delay time with a response to the presence of a change point and for the average time to a false alarm when a change point is detected using the CUSUM procedure.

UPPER AND LOWER BOUNDS FOR THE TAIL PROBABILITIES IN A BRANCHING RANDOM WALK WITH HEAVY-TAILED DISTRIBUTIONS OF JUMPS

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Let $\{\xi_{i,j}\}_{i,j\geq 1}$ be a family of independent random variables (r.v.) with common distribution F. We assume that F is centered, i.e.

$$\mathbb{E}\xi_{1,1} = 0,$$

and heavy-tailed, i.e.

$$\mathbb{E}e^{\lambda\xi_{1,1}} \equiv \int_{-\infty}^{\infty} e^{\lambda t} F(dt) = \infty$$

for all $\lambda > 0$.

Define a family of random walks $S_{i,n}$ as follows:

$$S_{i,0} = 0, \qquad S_{i,n} = \sum_{j=1}^{n} \xi_{i,j} \text{ for } n \geq 1.$$

Let Z be a positive integer-valued r.v. We study the tail distributional asymptotics for the following supremum:

$$R^g_{\mu,Z} = \max_{1\leq i\leq Z} \max_{0\leq n\leq \mu} (S_{i,n} - g(n)),$$

where $\mu \leq \infty$ is an arbitrary r.v. and g an arbitrary nonnegative function tending to infinity as $n \to \infty$.

We propose conditions under which the lower bound

$$\mathbb{P}\left(R^g_{\mu,Z} > x\right) \geq (1 + o(1))H^g_{\mu,Z}(x)$$

and the upper bound

$$\mathbb{P}\left(R^g_{\mu,Z} > x\right) \leq (1 + o(1))H^g_{\mu,Z}(x)$$

hold with uniformity over all suitable random time instances μ and functions g. Here

$$H^g_{\mu,Z}(x) = \sum_{n=1}^{\infty} \mathbb{E}\left[Z\mathbb{I}(\mu \geq n)\right] \overline{F}(x + g(n)).$$

Note that the model under consideration is a particular case of a branching random walk having branching only in the first generation.

The work is supported by Mathematical Center in Akademgorodok under agreement No. 075-15-2022-282 with the Ministry of Science and Higher Education of the Russian Federation.

CRITICAL GALTON-WATSON BRANCHING PROCESSES WITH A COUNTABLE SET OF PARTICLE TYPES AND RANDOM GRAPHS

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We consider genealogical trees of Galton-Watson branching processes and study the critical case corresponding to a one-vertex random tree with an independent, identically distributed number of edges for all vertices. The average number of edges coming out of a lower level vertex is 1. One of the fundamental theorems for these processes is a Yaglom-type theorem, which states that processes that do not degenerate to a distant time n contain at a given time the number of particles equal to this time n times exponentially distributed random variable. It is convenient to describe these conditional processes in terms of reduced trees, which are obtained from genealogical trees by eliminating subtrees that do not reach level n. A more complex model of Galton-Watson branching processes with a countable set of particle types, in which the types of descendants are obtained by summing the parent type with independent identically distributed multidimensional random variables, can be represented as trees with weights of edges and vertices defined above in the one-dimensional case.

We describe the averages and variances of a number of characteristics of reduced weighted trees, including the total weight of all vertices at a fixed level. We prove a number of limit theorems for reduced trees.

The work was carried out within the framework of the state task of the Institute of Mathematics of the Siberian Branch of the Russian Academy of Sciences, project FWNF-2022-0003.

ASYMPTOTIC EXPANSIONS FOR FIRST-PASSAGE TIMES OF AN OSCILLATING RANDOM WALK

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In this talk I shall consider asymptotic expansions for the tail of the distribution of the time when an oscillating random walk crosses a fixed level $-x \leq 0$ for the first time. Furthermore, I shall discuss a connection between such expansions and polyharmonic functions for killed random walks.

RANDOM POLYNOMIALS HAVING NO REAL ZEROS

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In their 2002 paper, Dembo, Poonen, Shao and Zeitouni obtained a power asymptotic n decreasing probability that a random polynomial of even degree n with i.i.d. coefficients has no real zeros. The exact power exponent was not found, but it was conjectured that it is equal to $-3/4$. Only in the summer of 2021 FitzGerald, Tribe, and Zaboronski posted the work in the arXiv with its proof. In this talk we will consider a similar problem for random polynomials whose coefficients have binomial variance. These polynomials were first considered by Kostlan, Shub, and Smale in their works in the early 90s of the last century.

PROBING HARMONY WITH ALGEBRA (AT LEAST, STATISTICALLY)

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In a recent statistical study, US researchers quantified attractiveness of a face by using measures of deviation from canonical "standards" like equality of eye width to interocular distance or golden ratio of nose to chin distance to nose width. The actual attractiveness formula is kept as a commercial secret, but using available published data we shall discuss if attractiveness is really a function of the geometry of a face and to which extent the harmony can be described by the algebra (even statistically). In the course of the talk we shall discuss the latest scientific results in the intersection of physiology, psychology, statistics and computer graphics on what attractiveness is and its biological roots.

POSTER TALKS

MODERATE DEVIATIONS PRINCIPLE FOR M-DEPENDENT RANDOM VARIABLES IN SUBLINEAR EXPECTATION SPACE

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The report will talk about the result obtained in the field of large deviations theory in nonlinear expectation space. The moderate deviations principle for a strictly stationary sequence of m-dependent random variables in a sublinear expectation space was obtained. Unlike known results, random variables are required to satisfy a less restrictive Cramer-like condition.

RESEARCH OF THE INFECTION MODEL IN QUEUES $M/M/K/0$

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We consider a queueing system with additional parameters designed to simulate the spread of infection. We discuss the dependence of the rate of infection on the number of people in the system and the effectiveness of quarantine depending on various disease parameters (percentage of the population infected, infectiousness, etc.).

DOUBLE BOOTSTRAP METHOD FOR TAIL INDEX ESTIMATION BY EXPECTILES

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Tail index estimation is an important topic in extreme value theory and has wide applications in financial and actuarial mathematics. Tail index in distribution defines the existence of high-order moments and allows estimating the decreasing rate of survival function. On the applied side of topic, such estimation gives opportunity to insure risky or extremal situations, which are modelled by power-law distributions.

There are plenty of approaches to estimate tail index. Most common is a group of estimators, which use properties of order statistics. Most well-known of them is a Hill estimator (Hill, [1975]). But the problem of such methods is their dependence from the number of selected order statistics k(n). For estimating the optimal value of k(n), a double bootstrap method was presented(Draisma, [1999]). Consistency and asymptotic normality holds, the representation of function k(n) is found.

Other methods for tail index estimation use so-called expectiles (Phillips, [2022]; Daouia, [2020]). Expectiles are solutions of the best weighted mean-squared predictor optimization problem and have similar properties to quantiles of distribution, but have even more: for example, expectile function is continuous, monotonically increasing function on [0,1]. Such methods like Hill-type estimators face the problem of searching for the optimal part of data from a sample.

The aim of this work is to explore the double bootstrap method and study the optimal level of subsample for tail index estimation by expectiles and statistical properties of such method. Also the aim is to implement a given method on Python programming language and compare it with existing double bootstrap method for Hill-type estimators.

MOMENTS OF THE FIRST DESCENDING EPOCH FOR A RANDOM WALK WITH NEGATIVE DRIFT

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We consider the first descending ladder epoch $\tau = \min\{n \geq 1 : S_n \leq 0\}$ of a random walk $S_n = \sum_1^n \xi_i, n \geq 1$ with i.i.d. summands having a negative drift $\mathbb{E}\xi = -a < 0$. Let $\xi^+ = \max(0, \xi_1)$. It is well-known that, for any $\alpha > 1$, the finiteness of $\mathbb{E}(\xi^+)^\alpha$ implies the finiteness of $\mathbb{E}\tau^\alpha$ and, for any $\lambda > 0$, the finiteness of $\mathbb{E}\exp(\lambda\xi^+)$ implies that of $\mathbb{E}\exp(c\tau)$ where $c > 0$ is, in general, another constant that depends on the distribution of ξ_1. We consider the intermediate case, assuming that $\mathbb{E}\exp(g(\xi^+)) < \infty$ for a positive increasing function g such that $\liminf_{x\to\infty} g(x)/\log x = \infty$ and $\limsup_{x\to\infty} g(x)/x = 0$, and that $\mathbb{E}\exp(\lambda\xi^+) = \infty$, for all $\lambda > 0$. Assuming a few further technical assumptions, we show that then $\mathbb{E}\exp((1-\varepsilon)g((1-\delta)a\tau)) < \infty$, for any $\varepsilon, \delta \in (0, 1)$.

STABILITY AND INSTABILITY OF A RANDOM MULTIPLE ACCESS SYSTEM WITH AN ENERGY HARVESTING MECHANISM

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The presentation will be about the results of the article by A. Rezler and M. Chebunin «Stability and instability of a random multiple access system with an energy harvesting mechanism». The article is based on the work by S. Foss, D. Kim, A. Turlikov «Stability and instability of a random multiple access model with adaptive energy harvesting», in which the regions of stability and instability of the model of a classical synchronized random multiple access system with one transmitting device, controlled by the ALOHA data transmission protocol and additionally equipped with an energy harvesting mechanism, were studied. Unlike the ordinary ALOHA system, in this model the authors assumed that each message has a battery that accepts one unit of energy, and only messages with a charged battery can be transmitted to the transmitting device. In addition to practical applications, this modification makes it possible to expand the stability region of the system. The main object of study in our work is a generalization of the above model, which is in the assumption that messages instead of a battery with a single unit of energy have a battery of unlimited capacity. The result of the work is a theorem on the preservation of the stability regions of the generalized model. Currently, we are working on a modification of the generalized model, which is in the fact that "old" charged messages lose their energy over time. The relevance of the modification appeared from the practical applications of the systems under study.

ASYMPTOTICS OF THE DISTRIBUTION OF THE EXIT TIME BEYOND A NON-INCREASING BOUNDARY FOR A COMPOUND RENEWAL PROCESS

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We investigate the probability that a homogeneous compound renewal process, which is also known as a continuous time random walk, is staying above a moving non-increasing boundary up to time T. As T tends to infinity, we find the exact asymptotics of this probability. We suppose that the jump size of the process under study has zero mean and finite variance whereas the renewal-time has a moment of the order of 3/2.

PROBABILITY APPROACH FOR GUESSING GAME IN RANDOM ENVIRONMENT

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Consider the following game: Let some question be asked, to which there are n possible answers $O_1, \dots, O_n$. In each round of the game, the correct answer to the question is chosen at random. The probability of each possible O_i response being correct is set at r_i $r_1 \geq r_2 \geq \cdots \geq r_n > 0$. Player $\mathcal{A}$ knows the question and probabilities of $r_1, \dots, r_n$. $\mathcal{B}$, apart from that, knows the right answer. $\mathcal{B}$ offers $\mathcal{A}$ to guess which answer is right, choosing from among the options. The $\mathcal{B}$ task is the right answer to pick a $k-1$ variant so that the probability of guessing $\mathcal{A}$ was the smallest. The $\mathcal{A}$ problem is to get a guessing algorithm out of k variants in which the probability of guessing is greatest.

The goals of this paper are: to build a mathematical model for this game, to obtain optimal mixed strategies for both players, as well as to find the probability of guessing the right answer, if players follow optimal mixed strategies.

The mathematical model of this problem is a zero-sum matrix game. In terms of game theory, it was possible to get the top and bottom price of the game, as well as the pure strategies of both players. What we can conclude is that there are no solutions in the pure strategies of the game. Unfortunately, this production is not computationally useful for mixed strategy and game pricing. The payment matrix has a dimension of $(C_{k-1}^{n-1})n \times k^{C{n_k}}$. The classical approach to solving matrix games involves solving the problem of linear programming with a –payment matrix constraint system. Since the matrix has a growth rate much higher than the exponential (by the number of possible responses), the solution is impossible to get even for small n and k. Within the framework of this paper, an approach to the construction of a system of equations of dimension much smaller than the original, based on the application of Bayesian theory, has been proposed.

ON THE DISTRIBUTION OF THE LENGTH OF THE SHORTEST PATH IN A GENERALISED BARAK-ERDŐS GRAPH

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We consider a random graph $\mathcal{G}_n$ that is an oriented version of Erdős-Renyi graph with set of vertices $\mathcal{V}_n = \{0, 1, 2, \ldots, n\}$ and random set of edges $\mathcal{E}_n \subset \mathcal{V}_n^2$. We assume that all the edges are directed from the smaller vertices to the larger. For $i < j$ let

$$p_{i,j}(n) := \mathbb{P}\left((i, j) \in \mathcal{E}_n\right).$$

Assume additionally that the events $\{(i, j) \in \mathcal{E}_n\}$ are mutually independent.

The graph $\mathcal{G}_n$ is called Barak-Erdős graph if $p_{i,j}(n)$ does not depend on the total number of vertices n and on vertices i and j. We study so-called generalised Barak – Erdős graph assuming $p_{i,j}(n)$ be a function of i, j and n.

Let L_n be the length of the shortest path between vertices 0 and n.

Assume that

$$p_{i,j}(n) = \frac{f(\frac{i}{n}, \frac{j}{n})}{n^\gamma},$$

where $f(x, y)$ – is a function Riemann-integrable on $[0, 1]^2$ and $\gamma \in (0, 1)$ is a positive constant.

The main result of our study is the following theorem.

Theorem. *Let $\gamma = 1 - 1/k$ for some $k \in \mathbb{N}$. Then*

$$\lim_{n\to\infty} \mathbb{P}(L_n = k + 1) = 1 - \lim_{n\to\infty} \mathbb{P}(L_n = k) = \exp\left(-c_k(f)\right),$$

where

$$c_k(f) = \int_{0<u_1<\ldots<u_{k-1}<1} \prod_{j=0}^{k-1} f(u_j, u_{j+1}) du_1 \cdots du_{k-1} \in [0, \infty],$$

$u_0 = 0$ *and* $u_k = 1$.

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MATHEMATICS CHALLENGES IN GENOME-WIDE ASSOCIATION STUDIES

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Genome-wide association studies (GWAS) is a field of studies with the goal of identifying genomic regions associated with development of human diseases. GWAS typically focus on the analysis of associations between the trait of interest and a large number of genetic markers dispersed evenly throughout the genome.

This presentation is dedicated to relevant problems in the GWAS field, as well as directions for developing more accurate GWAS models.

DISTRIBUTION OF THE LENGTH AND THE HEIGHT OF YHE REGENERATION CYCLE FOR A RANDOM WALK WITH DRIFT

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Consider a simple random walk with jumps of $+1$ and -1 and non-zero drift provided that its trajectory does not take negative values within the infinite time interval. Such a random walk has a regenerative structure generated by the time instants when all previous values are smaller and all future values are not smaller than the current one. We analyse the distributions of the length and the height of a typical regenerative cycle of the process. We show that this analysis may be reduced to a combinatorial problem of finding the so-called "indecomposable" paths. We give recursive relations that allow us to calculate the distributions of the length and the height of the regeneration cycle.

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