

# “Complex Geometry in Byurakan”

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## Organizers

Steklov Mathematical Institute of Russian Academy of Sciences, Moscow

Steklov International Mathematical Center, Moscow

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# Complex Geometry in Byurakan

## Abstracts

**Ekaterina Amerik**

**Title:** On algebraically coisotropic submanifolds

**Abstract.** This is a joint work with F. Campana. Recall that a submanifold  $X$  in a holomorphic symplectic manifold  $M$  is said to be coisotropic if the corank of the restriction of the holomorphic symplectic form  $s$  is maximal possible, that is equal to the codimension of  $X$ . In particular a hypersurface is always coisotropic. The kernel of the restriction of  $s$  defines a foliation on  $X$ ; if it is a fibration,  $X$  is said to be algebraically coisotropic. A few years ago we proved that a non-uniruled algebraically coisotropic hypersurface  $X \subset M$  is a finite etale quotient of  $C \times Y \subset S \times Y$ , where  $C \subset S$  is a curve in a holomorphic symplectic surface, and  $Y$  is arbitrary holomorphic symplectic. We prove some partial results on the higher-codimensional analogue of this, with emphasis on the abelian case.

**Grigory Mikhalkin**

**Title:** Exoteric symplectic capacities and algebraic curves

**Abstract:** Symplectic capacities are numerical characteristics providing necessary conditions for embeddings of one symplectic domain into another. The talk will present a down-to-earth elementary construction of series of symplectic capacities based on "exoteric" curves, i.e. those coming from the outside.

In a number of cases (thanks to Gromov's compactness), exoteric capacities can be easily computed by means of (closed) algebraic curves on projective varieties. These considerations turn out to be sufficient to reconstruct the famous "Fibonacci staircase" theorem of McDuff and Schlenk, governing symplectic squeezings of ellipsoids into the 4-ball. In particular, the Seiberg-Witten theory is no longer needed for the proof of this theorem.

**Dmitrii Pirozhkov**

**Title:** Categorical Torelli theorem for hypersurfaces

**Abstract:** Bondal and Orlov proved that any Fano variety can be reconstructed from its derived category of coherent sheaves. If  $X$  is a Fano hypersurface in a projective space, then its derived category has an interesting subcategory referred to as a "Kuznetsov component" or "residual category". Huybrechts and Rennemo showed that this subcategory equipped with a certain autoequivalence determines  $X$  uniquely if the degree of  $X$  divides  $\dim(X) + 2$ . I will explain the generalization of their theorem that does not require any divisibility conditions.

**Nikon Kurnosov****Title:** Construction of BG-manifolds

**Abstract:** I will review the construction of BG-manifolds and their algebraic reduction. Their deformation theory is quite similar to the deformation theory of hyperkahler manifolds, what leads to the interest to study this class of non-Kahler holomorphic symplectic manifolds. The talk is based on joint works with Bogomolov, Kuznetsova, Verbitsky, Yasinsky and some works in progress.

**Vadim Vologodsky****Title:** On the  $G_m^\sharp$  action on the category of  $p$ -torsion crystals.

**Abstract:** Using his concept of prismatization Drinfeld explained (in unpublished but widely circulated draft) how a lifting of a smooth variety  $X$  over a perfect field  $k$  of characteristic  $p$  to  $W_2(k)$  yields an action of the divided power envelope  $G_m^\sharp$  of  $G_m$  on the de Rham complex of  $X$ . In this talk I will prove that the stack of  $G_m^\sharp$ -actions on the category of  $p$ -torsion crystals on  $X$  that respect the monoidal structure and rescale the  $p$ -curvature is a gerbe under the tangent sheaf  $T_X$  equivalent to the gerbe of liftings of  $X$  over  $W_2(k)$ .

**Alexandra Kuznetsova****Title:** Automorphisms of Bogomolov-Guan manifolds

**Abstract:** Bogomolov-Guan manifolds are the only known examples of holomorphic symplectic, simply connected smooth and at the same time non-Kahler manifolds. They have a complicated construction but one can study their groups of automorphisms using the notion of the algebraic reduction. I will talk about the structure of special fibers of their algebraic reductions and I will use this to show that the group of regular automorphisms of Bogomolov-Guan manifolds is Jordan and in the four-dimensional case the same is true for the group of bimeromorphic automorphisms.

**Yuri Prokhorov****Title:** Singular Del Pezzo varieties.

**Abstract.** A del Pezzo variety  $X$  is a Fano variety whose anticanonical class has the form

$$-K_X = (n - 1)A,$$

where  $A$  is an ample line bundle and  $n$  is the dimension of  $X$ . This is a higher dimensional analog the notion of del Pezzo surfaces. I am going to discuss biregular and birational classifications of del Pezzo varieties admitting terminal singularities.

The talk is based on a joint work with Alexander Kuznetsov (in preparation).

**Dmitry Krekov****Title:** A two-dimensional height pairing for Chow motives

**Abstract:** I will introduce the notion of abstract height pairing for Chow motives, which axiomatizes the properties of height pairing constructed by Beilinson

and Bloch for algebraic varieties over function fields and number fields. Given a field  $K$  with an abstract height pairing (with certain restrictions in case of positive characteristic) on the category of Chow motives over  $K$  and an extension  $L$  of  $K$  of transcendence degree 1, I will define a pairing on a full subcategory of Chow motives of  $L$ , which arise as the base change of certain Chow motives over  $K$ . This pairing is a two-dimensional analogue of height pairing. If time permits, I will compute some explicit examples. The definition and necessary properties of Chow motives will be recalled.

### **Vardan Oganessian**

**Title:** Lagrangian Delzant theorem and its applications

**Abstract:** Let  $(M, \omega)$  be a symplectic manifold and  $L$  be a Lagrangian submanifold with the Maslov class  $m$ . The Lagrangian  $L$  is called monotone if  $\omega = km$ , where  $k$  is some positive real constant. Monotone Lagrangian submanifolds are of special interest due to their prominent role in Floer theory. Unfortunately, it is very hard to construct explicit examples of monotone submanifolds, even for  $M = \mathbb{C}^n, \mathbb{C}\mathbb{P}^n$ .

In this talk we present a new effective method for constructing monotone Lagrangian submanifolds. We associate a closed Lagrangian submanifold  $L$  of  $\mathbb{C}^n$  and  $\mathbb{C}\mathbb{P}^n$  to each Delzant polytope  $P$ . We prove that  $L$  is monotone if and only if  $P$  is Fano. Moreover, our method allows us to compute the Floer homology of the Lagrangians. Also, we show that the new method can be used to study the singular cohomology ring of real toric spaces.

### **Bogdan Zavyalov**

**Abstract:** Lefschetz Theorems in flat cohomology.

**Title:** I will discuss Lefschetz type results for flat cohomology of finite flat group schemes over a field of characteristic  $p$ . As a by-product, I will show how to get a simplified proof of a recent result of Cesnavicius and Scholze that Picard group of a (possibly singular) complete intersection surface is torsion-free.

Joint work with Sean Cotner.

### **Costya Shramov**

**Title:**  $p$ -Jordan property for automorphism groups.

**Abstract:** I will discuss the so-called  $p$ -Jordan property which is suitable to measure the complexity of infinite groups of geometric origin, e.g. automorphism groups of algebraic varieties over fields of characteristic  $p$ .

### **Andrey Soldatenkov**

**Title:**  $\mathbb{C}$ -symplectic structures and Lagrangian fibrations

**Abstract:** I will talk about our joint work with M. Verbitsky. A  $\mathbb{C}$ -symplectic structure is a complex-valued 2-form that becomes holomorphically symplectic with respect to an appropriate complex structure. One can prove an analogue of Moser's

lemma for  $\mathbb{C}$ -symplectic structures. I will explain how one can use this to study holomorphic Lagrangian fibrations and state some open problems.

### Grigory Papayanov

**Title:** Fedosov quantization and the period map.

**Abstract:** Deformation quantizations of symplectic varieties were classified by Fedosov in the smooth case, by Nest-Tsygan in the complex analytic case and by Kaledin-Bezrukavnikov for algebraic varieties over the field of characteristic zero. The common answer in all three context is that, under the condition the Hodge filtration on the de Rham cohomology splits, is that the set of the equivalence classes of quantizations is isomorphic to  $H^2(\Omega_{dR}^{\geq 1}[[\hbar]])$ , and this set is naturally non-linearly embedded into  $H^2(\Omega_{dR}[[\hbar]])$  via the so-called period map. We propose the upgrade of the period map from the map of sets to the transformation between appropriately defined deformation functors, which a) gives an alternative proof of the Fedosov-Nest-Tsygan-Kaledin-Bezrukavnikov theorems and b) allows one, in principle, to write down a formula for the period map.

### Andrey Trepalin

**Title:** Classification of pointless del Pezzo surfaces of degree 8

**Abstract:** Let  $k$  be an algebraically nonclosed field of characteristic 0. We will describe biregular and birational classification of pointless del Pezzo surfaces of degree 8 in terms of the Brauer group  $Br(k)$ . Moreover, we describe minimal surfaces, that are birationally equivalent to a given pointless del Pezzo surface of degree 8. In particular, we describe birationally rigid del Pezzo surfaces of degree 8.

### Marat Rovinsky

**Title:** Invariant fields of rational functions and semilinear representations of symmetric groups over them

**Abstract:** Let  $k$  be a field,  $S$  be an infinite set,  $k(S)$  be the field of rational functions over  $k$  in the variables labeled by  $S$ ,  $F_S$  a slightly more general field (depending on a field  $F$  of algebraic functions over  $k$  of one variable). The group  $G$  of all permutations of the set  $S$  acts on the field  $F_S$  (as well as on  $k(S)$ ). The aim of the talk is to describe smooth semilinear representations of the group  $G$  over some  $G$ -invariant subfields  $K$  in  $F_S|_k$

In particular, unlike the linear representations, one can construct irreducible non-trivial semilinear representations of dimensions 1, 2, 3, 4, 5 for appropriate  $K$  in  $k(S)$ .

The transcendence degree  $d$  of  $F_S|_K$  over any  $G$ -invariant subfield  $K$  in  $F_S|_k$  is finite. If  $d > 1$ , the characteristic is 0 and  $K$  is algebraically closed in  $F_S|_k$ , then  $d = 2$  or 3 and the isomorphism class of  $K$  depends only on  $d$  and  $k$ . To each one-dimensional algebraic  $k$ -group admitting an  $F$ -rational generic point, one can associate a class of  $G$ -invariant subfields in  $F_S|_k$  with  $d = 1$ . In characteristic 0, it is possible to show the converse by an analytic argument.

**Dmitrii Kubrak****Title:** de Rham cohomology of  $BSpin_n$  over  $F_2$ 

**Abstract:** I will explain how to explicitly compute the de Rham cohomology ring of  $BSpin_n$  over  $F_2$  using the theory of prismatic cohomology of Bhatt-(Morrow)-Scholze. The main idea behind the computation is that in char 2 the Breuil-Kisin module structure can help to fully understand the (mod 2)-prismatic cohomology by only looking at its étale specialization. The key ingredients that then go in the computation are the prismatic Hochschild-Serre spectral sequence (j. with F.Scavia), étale comparison for prismatic cohomology in the stacky setting (j. with A.Prikhodko) and a certain commutative algebra computation using Groebner basis. This allows to extend the computations made by B.Totaro and E.Primožic for  $n \leq 11$  to all  $n$ . Joint work with A.Chua.

**Gregory Taroyan****Title:** Automorphisms of moment-angle manifolds

**Abstract:** I will explain how to compute automorphisms of a large class of non-Kähler manifolds known as moment-angle manifolds. In particular, I will describe automorphism groups of Hopf and Calabi—Eckmann manifolds.