International conference “Nonlinear waves and Frobenius structures in geometry and physics” dedicated to the memory of Boris Dubrovin

Schedule

Steklov Mathematical Institute of Russian Academy of Sciences, Moscow
Steklov International Mathematical Center, Moscow

The conference is supported by the Simons Foundation and the Ministry of Science and Higher Education of the Russian Federation (the grant to the Steklov International Mathematical Center, agreement no. 075-15-2019-1614).

Wednesday, November 17

All times are indicated using the GMT+3 Moscow timezone (+11 hours from PST, +8 hours from EST). All the talks in-person will be held at Steklov Mathematical Institute, room 104, and streamed to Zoom. All online talks will be shown in room 104 as well.

11:00–11:20	**Opening**		
11:20–12:00	104	**Sergei Novikov** Steklov Mathematical Institute, Moscow	Наша совместная работа с Борисом Дубровиным (talk in Russian)
12:00–12:30	**Coffee**		
12:30–13:10	Zoom	**Alexandr Buryak** Higher School of Economics, Moscow	The Dubrovin-Zhang systems and relations in the cohomology of the moduli spaces of curves
13:20–14:00	Zoom	**Tamara Grava** SISSA, Italy	Gibbs ensemble for Integrable Systems, a case study: the discrete nonlinear Schrödinger equation
14:00–16:00	**Lunch**		
16:00–16:40	104	**Dmitry Orlov** Steklov Mathematical Institute, Moscow	Exceptional collections, mirror symmetry, and Dubrovin’s conjecture
16:50–17:30	Zoom	**Vasily Golyshev** IITP RAS, Moscow	From Dubrovin’s conjectures to motivic gamma functions
17:30–18:00	**Coffee**		
18:00–18:40	Zoom	**Alexander Varchenko** University of North Carolina at Chapel Hill, USA	Frobenius-like structures of arrangements of hyperplanes
19:00-19:40	Zoom	**Alexander Givental** University of California, Berkeley, USA	On K-theory of Deligne-Mumford spaces

International conference “Nonlinear waves and Frobenius structures in geometry and physics” dedicated to the memory of Boris Dubrovin

Schedule

Thursday, November 18

All times are indicated using the GMT+3 Moscow timezone (+11 hours from PST, +8 hours from EST). All the talks in-person will be held at Steklov Mathematical Institute, room 104, and streamed to Zoom. All online talks will be shown in room 104 as well.

11:00–11:40	Zoom	**Maxim Smirnov** University of Augsburg, Germany	Quantum cohomology of coadjoint varieties
11:40–12:10	**Coffee**		
12:10–12:50	104	**Alexander Kuznetsov** Steklov Mathematical Institute, Moscow	Quantum cohomology and derived categories
13:00–13:40	104	**Alexey Basalaev** Higher School of Economics, Moscow	Integrable systems associated to infinite series of Dubrovin-Frobenius manifolds
13:40–15:30	**Lunch**		
15:30–16:10	Zoom	**Andrey Mironov** Sobolev Institute of Mathematics, Novosibirsk	Discretization of Baker-Akhiezer modules and commuting difference operators in several discrete variables
16:20–17:00	Zoom	**Evgeny Ferapontov** Loughborough University, UK	Second-order PDEs in 3D with Einstein-Weyl conformal structure
17:00–17:30	**Coffee**		
17:30–18:10	Zoom	**Vladimir Dragović** University of Texas at Dallas, USA	Chebyshev dynamics, isoharmonic deformations, and constrained Schlesinger systems

International conference “Nonlinear waves and Frobenius structures in geometry and physics” dedicated to the memory of Boris Dubrovin

Schedule

Friday, November 19

All times are indicated using the GMT+3 Moscow timezone (+11 hours from PST, +8 hours from EST). All the talks in-person will be held at Steklov Mathematical Institute, room 104, and streamed to Zoom. All online talks will be shown in room 104 as well.

11:00–11:40	104	**Sergei Lando** HSE & Skoltech, Moscow	Real Hurwitz numbers
11:40–12:10	**Coffee**		
12:10–12:50	104	**Maxim Kazarian** HSE & Skoltech, Moscow	Topological recursion for generalized Hurwitz numbers
13:00–13:40	104	**Mikhail Feigin** University of Glasgow, UK	Trigonometric solutions of WDVV and related equations
13:40–15:30	**Lunch**		
15:30–16:10	104	**Oleg Mokhov** Moscow State University, Moscow	Curved WDVV equations and the theory of submanifolds in pseudo-Euclidean spaces
16:20–17:00	104	**Maxim Pavlov** Lebedev Physical Institute, Moscow	New Hamiltonian formalism for semi-hamiltonian systems of hydrodynamic type
17:00–17:30	**Coffee**		
17:30–18:10	Zoom	**Marco Bertola** Univerity Concordia, Canada	KP τ-functions and biorthogonality on a Riemann surface
18:30–19:10	Zoom	**Iskander Taimanov** Sobolev Institute of Mathematics, Novosibirsk	Creation of singularities of 2D soliton equations represented by L, A, B –triples

Talk abstracts

Organizers

Steklov Mathematical Institute of Russian Academy of Sciences, Moscow

Steklov International Mathematical Center, Moscow

Wednesday Nov 17th

Наша совместная работа с Борисом Дубровиным **(talk in Russian)**

Sergei Novikov

Steklov Mathematical Institute, Moscow

The Dubrovin-Zhang systems and relations in the cohomology of the moduli spaces of curves

Alexandr Buryak

Higher School of Economics, Moscow

A far-reaching generalization of Witten's conjecture suggested by Dubrovin and Zhang says that the generating series of correlators of an arbitrary semisimple cohomological field theory (CohFT) is a solution of a certain system of PDEs of very special type. This result was proved by Posthuma, Shadrin, and the speaker, however there are good reasons to believe that the Dubrovin-Zhang conjecture is true for nonsemisimple CohFTs and also for certain generalizations of the notion of a CohFT. In the talk I would like to show how certain relatively elementary arguments allow to propose a system of conjectural relations in the cohomology of the moduli spaces of curves, which implies the Dubrovin-Zhang conjecture in the maximal generality. A part of these relations is proved. The talk is based on a joint work with Sergey Shadrin.

Gibbs ensemble for Integrable Systems, a case study: the discrete nonlinear Schrödinger equation

Tamara Grava

SISSA, Italy

We consider the discrete defocusing nonlinear Schrodinger equation in its integrable version, that is called Ablowitz Ladik lattice. We consider the Gibbs ensemble for the Ablowitz Ladik lattice. In this setting the Lax matrix of the Ablowitz Ladik lattice turns into a random matrix that is related to the circular beta-ensemble at high temperature. We obtain the density of states of the random Lax matrix, when the size of the matrix goes to infinity, by establishing a mapping to the one-dimensional log-gas. The density of states is obtained via a particular solution of the double-confluent Heun equation. Joint work with Guido Mazzuca.

Exceptional collections, mirror symmetry, and Dubrovin'sconjecture

Dmitry Orlov

Steklov Mathematical Institute, Moscow

From Dubrovin's conjectures to motivic gamma functions

Vasily Golyshev

IITP RAS, Moscow

I will give a survey of some problems and recent results motivated by Dubrovin's original conjectures on the monodromy of structure connections on certain Frobenius manifolds.

Frobenius-like structures of arrangements of hyperplanes

Alexander Varchenko

University of North Carolina at Chapel Hill,USA

We consider the algebra of functions on the critical set of the master function of a weighted arrangement of hyperplanes in k-dimensional complex affine space. We construct a potential function of variables, labeled by hyperplanes of the arrangement, such that the structure constants of the algebra multiplication are given by the $2k+1$-st derivatives of the potential function. The presence of this potential function is a manifestation of a Frobenius-like structure similar to the Frobenius manifold structure.

On K-theory of Deligne-Mumford spaces

Alexander Givental

University of California, Berkeley, USA

I will discuss the state of affairs in the theory of K-theoretic Gromov-Witten invariants of the point.

Thursday Nov 18th

Quantum cohomology of coadjoint varieties

Maxim Smirnov

University of Augsburg, Germany

Quantum cohomology of homogeneous spaces G/P has been studied extensively since the 90's. In this talk I will concentrate on some recent structural results about small and big quantum cohomology for a special class of homogeneous spaces — the so called coadjoint varieties. The talk is based on a joint work in progress with Nicolas Perrin.

Quantum cohomology and derived categories

Alexander Kuznetsov

Steklov Mathematical Institute, Moscow

In the talk I will try to explain how the structure of the small quantum cohomology ring of a smooth projective variety constrains its derived category of coherent sheaves.

Discretization of Baker-Akhiezer modules and commuting difference operators in several discrete variables

Andrey Mironov

Sobolev Institute of Mathematics, Novosibirsk

We introduce the notion of discrete Baker-Akhiezer (DBA) modules, which are modules over the ring of difference operators, as a certain discretization of Baker-Akhiezer modules which are modules over the ring of differential operators. We use it to construct commuting difference operators

with matrix coefficients in several discrete variables. The results were obtained with Atsushi Nakayashiki.

Second-order PDEs in 3D with Einstein-Weyl conformal structure

Evgeny Ferapontov

Loughborough University, UK

Einstein-Weyl geometry is a triple (D, g, w) where D is a symmetric connection, g is a conformal structure and w is a covector such that

1. connection D preserves the conformal class g,

2. trace-free part of the symmetrised Ricci tensor of D vanishes.

Three-dimensional Einstein-Weyl structures naturally arise on solutions of second-order dispersionless integrable PDEs in 3D. In this context, g coincides with the characteristic conformal structure and is therefore uniquely determined by the equation. On the contrary, covector w is a somewhat more mysterious object, recovered from the Einstein-Weyl conditions.

It turns out that, for generic second-order PDEs (for instance, for all equations not of Monge-Ampere type), the covector w is also expressible in terms of the equation, thus providing an efficient "dispersionless integrability test". The knowledge of g and w provides a dispersionless Lax pair by an explicit formula which is apparently new.

Some partial classification results of PDEs with Einstein-Weyl characteristic conformal structure are obtained. A rigidity conjecture is proposed according to which for any generic second-order PDE with Einstein-Weyl property, all dependence on the 1–jet variables can be eliminated via a suitable contact transformation.

This talk is based on Joint work with S. Berjawi, B. Kruglikov and V. Novikov; `arXiv:2104.02716`.

Chebyshev dynamics, isoharmonic deformations, and constrained Schlesinger systems

Vladimir Dragović

University of Texas at Dallas, USA

The talk is based on interrelations between integrable billiards, extremal polynomials, Riemann surfaces, potential theory, and isomonodromic deformations. We discuss injectivity properties of rotation and winding numbers. We study dynamics of Chebyshev polynomials on several intervals and introduce a notion of iso-harmonic deformations. We study their isomonodromic properties and formulate a new class of constrained Schlesinger systems. We provide explicit solutions to

these systems. The talk is based on joint results with Vasilisa Shramchenko, including work in progress.

Friday Nov 19th

Real Hurwitz numbers

Sergei Lando

HSE & Skoltech, Moscow

In the complex case, Hurwitz numbers enumerate meromorphic functions with a given set of critical values, ramification over each being a prescribed partition of the degree of the function. Hurwitz numbers do not depend on the specific positions of the critical values. Simple Hurwitz numbers enumerate meromorphic functions with a given set of critical values, ramification over one of which is a prescribed partition of the degree of the function, while all the other critical values are simple. Simple Hurwitz numbers play a crucial role in the study of intersection theory on moduli spaces of algebraic curves (ELSV, 1999). They define a topological field theory (associated to symmetric groups) and, when collected into an appropriate generating function, satisfy a so-called cut-and-join relation and form a τ-function to the Kadomtsev–Petviashvili integrable hierarchy of partial differential equations (Okounkov 2000).

Understanding of Hurwitz numbers in the real case is much more poor. Real Hurwitz numbers enumerate real meromorphic functions on real algebraic curves of given genus. There are several versions of these numbers, depending on, for example, whether we presume that the real curve is separating or not. Simple real Hurwitz numbers share certain properties of complex ones. They define topological field theories, and the generating functions for them are subject to generalized cut-and-join equations. However, no relation to the topology of moduli spaces of real algebraic curves and/or integrable hierarchies is known. In addition, while in the complex case the cut-and-join operator is diagonalizable in the basis of Schur polynomials, there is no such statement in the real one.

The general case, when we consider Hurwitz numbers that are not necessarily simple, is even more intricate. In this case the number of meromorphic functions depends on the mutual positions of the ramification points on the real line. Itenberg and Zvonkine (2018) managed to assign positive or negative sign to polynomials making the resulting signed enumeration independent of the mutual positions of the ramification points. No way to extend their definition to the case of general real meromorphic functions is known.

The talk is based on the papers below.

S. Natanzon, *Simple Hurwitz numbers of a disk*, Functional Analysis and Its Applications, March 2010, Volume 44, Issue 1, pp 36–47

M. Kazarian, S. Lando, S. Natanzon, *On framed simple purely real Hurwitz numbers*, Izvestiya Mathematics, 2021. Vol. 8

E. Krasilnikov, *Efficient computation of real Hurwitz numbers*, submitted

Topological recursion for generalized Hurwitz numbers

Maxim Kazarian

HSE & Skoltech, Moscow

The topological recursion or Chekhov-Eunard-Orantin recursion is an inductive procedure for an explicit computation of correlator functions appearing in a large number of problems in mathematical physics, from matrix integrals and Gromov-Witten invariants to enumerations of maps and meromorphic functions with prescribed singularities. In spite of existence of a huge number of known cases where this procedure does work, its validity and universality still remains mysterious in much extent.

We develop a new tool based on the theory of KP hierarchy that allows one not only formally prove it in a unified way for a wide class of problems but also to understand its true nature and the origin. These problems include enumeration various kinds of Hurwitz numbers: ordinary ones, orbifold, double, monotone, r-spin Hurwitz numbers, as well as enumeration of (hyper) maps and extends much beyond. The talk is based on a joint work with B.Bychkov, P.Dunin-Barkowski, S.Shadrin.

Trigonometric solutions of WDVV and related equations

Mikhail Feigin

University of Glasgow, UK

I am going to discuss a class of solutions of WDVV equations which are constructed in terms of special collections of vectors with multiplicities. They can be viewed as the trigonometric version of a class of rational solutions introduced by Veselov about 20 years ago. In the case of root systems rational solutions are almost dual to Frobenius manifold structure on the space of orbits of the Coxeter group. The class of rational solutions is closed under the natural operations of taking subsystems and restrictions. We show that similar operations can be applied in the trigonometric settings. In the case of root systems trigonometric solutions are expected to be almost dual to Frobenius manifold structures on the orbit spaces of the extended affine Weyl groups, and in the simply-laced cases they are known to describe quantum cohomology of the resolutions of simple singularities.

I am also going to discuss a very close relation between WDVV equations and the commutativity equations $F_iF_j = F_jF_i$. These equations appeared in the supersymmetric mechanics and they also admit special trigonometric (and rational) solutions.

The talk is based on joint works with M. Alkadhem.

Curved WDVV equations and the theory of submanifolds in pseudo-Euclidean spaces

Oleg Mokhov

Moscow State University, Moscow

Earlier it was proved by the author that the WDVV equations are natural special reductions of the fundamental equations of the theory of submanifolds in pseudo-Euclidean spaces and any Frobenius manifold can be realized as a flat submanifold with flat normal bundle in pseudo-Euclidean space. In this talk we consider the curved WDVV equations arising in some physical models and prove that they are also natural special reductions of the fundamental equations of the theory of submanifolds in pseudo-Euclidean spaces. Besides, we consider geometry of a special class of submanifolds in pseudo-Euclidean spaces, namely, submanifolds with potential of normals.

This research was supported by the Russian Science Foundation under grant 20-11-20214.

New Hamiltonian formalism for semi-hamiltonian systemsof hydrodynamic type

Maxim Pavlov

Steklov Mathematical Institute, Moscow

In 1983 Boris A. Dubrovin and Sergei P. Novikov introduced the differential geometric Poisson bracket of the first type.

In terms of Riemannian (pseudo-Riemannian) geometry this means that a given system of hydrodynamic type defines a flat metric computed using the Tsarev lemma for a given affinor. (The velocity matrix which is a $(1,1)$–tensor is called an affinor.)

In the case of semi-hamiltonian (i.e. integrable using the generalized Tsarev hodograph method) hydrodynamic type systems the existence of such a Poisson bracket leads to the existence of a first-order differential operator translating the solutions of one system of linear equations (corresponding to the densities of the conservation laws) to another linear system (corresponding to the commutating flows).

We will present here a construction based on a map from the solutions of the second linear system to solutions of the first. In this case there also exists a first-order differential operator with another corresponding Poisson bracket.

Moreover, in this case the hydrodynamic type systems have a non-local Hamiltonian formalism though they admit a local Langrangian form.

We will discuss examples and a Egorov reduction generating an infinite number of local Hamiltonian structures.

KP τ-functions and biorthogonality on a Riemann surface

Marco Bertola

Univerity Concordia, Canada

We will discuss a generalization of the construction by Krichever of algebro-geometric solutions of the KP hierarchy. The generalization requires additional data consisting in an arbitrary measure on a path embedded on a Riemann surface and a related notion of biorthogonality of sections of degree zero line bundles.

The latter notion is also related to a novel construction of Padé approximation of Weyl-Stieltjes function on a Riemann surface, which I will discuss if time permits.

Creation of singularities of 2D soliton equations represented by L, A, B—triples

Iskander Taimanov

Sobolev Institute ofMathematics, Novosibirsk
