International conference «Integrability»
Dedicated to 75th Anniversary of A. K. Pogrebkov

September 22 – 24, 2021

Organizers

Steklov Mathematical Institute of Russian Academy of Sciences, Moscow

Steklov International Mathematical Center, Moscow

HSE – Skoltech International Laboratory of Representation Theory and Mathematical Physics, Moscow

The conference is supported by the Simons Foundation and
the Ministry of Science and Higher Education of the Russian Federation
(the grant to the Steklov International Mathematical Center,
agreement no. 075-15-2019-1614).

Abstracts

Alexander Belavin
(Landau Institute for Theoretical Physics of RAS)

Explicit construction of $N = 2$ superconformal orbifolds

Following Gepner's approach, we propose a construction of models of tensor product orbifolds of Minimal models of two-dimensional Field Theory with $N = 2$ superconformal symmetry. To build models that satisfy the requirements of modular invariance, our construction uses a spectral flux transformation. We demonstrate this construction with a specific example and show that its application ensures the modular invariance of the partition function simultaneously with the mutual locality of the fields of the theory under consideration.

Francesco Calogero
(Physics Dept., University of Rome "La Sapienza")

Recent identifcations of explicitly solvable systems of nonlinear Ordinary Differential Equations (ODEs)

After a terse discussion of the various possible significances of the term "solvable", I will review a few recent results concerning systems of nonlinear Ordinary Differential Equations, such as special cases of the prototypical system

$$\dot{x}_n(t) = \sum_{m_1,m_2=1,\ m_1\geq m_2}^{N} [c_{nm_1m_2} x_{m_1}(t)\, x_{m_2}(t)] \ , \quad n = 1, 2, ..., N \ ,$$

and of analogous systems featuring *polynomial* right-hand sides (possibly *non homogeneous* and of *higher degree* than 2), or *homogeneous* but *nonpolynomial* right-hand sides.

Almost all the results reported in this talk have been obtained in collaboration with Prof. Farrin Payandeh (Physics Dept., Payame Noor University in Tabriz, Iran).

Leonid Chekhov
(Steklov Mathematical Institute)

Integrable structures on directed networks

Sergey Dobrokhotov
(Ishlinsky Institute for Problems in Mechanics of Russian Academy of Sciences)

The Carrier-Greenspan transformation, its modifications and application in problems about waves in basins with gentle shores.

The Carrier-Grispan transformation reduces the problem of constructing solutions to a one-dimensional nonlinear shallow water system over a linear bottom to a one-dimensional wave equation with a velocity $c^2(x) = x$. We discuss the modifications of this transformation and applications in one-dimensional and two-dimensional problems about waves in basins with gentle shores. The solutions of the problem of standing waves in long basins obtained with the help of such a modification are also compared with the experiment. This work was done together with V. A. Kalinichenko, D. S. Minenkov and V. E. Nazaykinskii and supported by the Russian National Science Foundation (grant 21-11-00341).

Vladimir Gerdjikov
(Institute of Mathematics and Informatics, Bulgarian Academy of Sciences)

Recursion operators and the hierarchies of MKdV equations related to Kac-Moody algebras

Gerard Helminck
(KdV Institute University of Amsterdam)

Darboux transformations of integrable hierarchies in the pseudo-difference operators

In the talk we present first two varieties from which one can construct solutions of the dKP hierarchy and its strict version. Next we describe what Darboux transformations are in the pseudo difference context and which two points in each variety are connected by such a transformation. We conclude with giving an explicit form of the operators that perform these transformations.

Boris Konopelchenko
(Dipartimento di Matematica e Fisica "Ennio De Giorgi", Università el Salento)

Gradient catastrophes for the multidimensional homogeneous Euler equation

Sotiris Konstantinou-Rizos
(Centre of Integrable Systems, P.G. Demidov Yaroslavl State University)

Darboux transformations and Zamolodchikov's tetrahedron equation

This talk is concerned with the construction of new tetrahedron maps, namely solutions to Zamolodchikov's functional tetrahedron equation and parametric tetrahedron equation. We present a method for constructing such maps via Darboux transformations. In particular, we study matrix refactorisation problems for Darboux matrices associated with the nonlinear Schrödinger (NLS) and the derivative nonlinear Schrödinger (DNLS) equation, and we construct novel nine-dimensional tetrahedron maps. We show that the latter can be restricted to six-dimensional parametric tetrahedron maps on invariant leaves. Finally, we construct parametric tetrahedron maps employing degenerated Darboux transformations of NLS and DNLS type.

This work was supported by the Russian Science foundation (grant No. 20-71-10110).

Evgeny Kuznetsov
(Lebedev Physical Institute of RAS)

Breaking of slipping flows in the boundary layer

The mixed Lagrangian-Eulerian description is applied to the inviscid Prandtl equation. This description for two-dimensional flows coincides with the application of the so-called Crocco transformation. For the zeroth value of the pressure gradient, the velocity component u parallel to the wall for the Prandtl equation plays the role of Lagrangian coordinate and the second - Eulerian - variable coincides with the Cartesian coordinate x along the wall. In new variables x, u, t the continuity equation yields linear relation between stream function and the coordinate $y = y(x, u, t)$ which defines the level line $u = \text{const}$. This allows constructing the exact solution in the implicit form by introducing the generating function. It is shown that in the case of the slipping boundary condition the singularity formation of the gradient catastrophe type takes place at the wall due to the compressible character of the flow there. The same character for the singularity formation remains for arbitrary dependence of pressure on x. The velocity gradient near the singular point behaves like $(t_0 - t)^{-1}$ where t_0 is the collapse time. Such singularity formation occurs also for the three-dimensional Prandtl equation in the case of the slipping boundary condition at the wall. The breaking, in this case, generates the singular vorticity perpendicular to the wall.

Franco Magri

A new tale of an old story

I will try to present a few amazing outcomes of a new reading of one of the brightest paper of the theory of integrable systems written in the past: the paper by Sophie Kowalewski on the theory of integrable tops.

Andrei Marshakov
(Skoltech)

Quivers, clusters, refactorization and deautonomization

Alexander Mikhailov
(University of Leeds)

Supersymmetric localization on Lie groups

In my talk I'll discuss a new approach to the problem of quantisation of dynamical systems, introduce the concept of quantisation ideals and provide meaningful examples. Traditional quantisation theories start with classical Hamiltonian systems with variables taking values in commutative algebras and then study their non-commutative deformations, such that the commutators of observables tend to the corresponding Poisson brackets as the (Planck) constant of deformation goes to zero. I am proposing to depart from Free associative mechanics - i.e. dynamical systems defined on a free associative algebra $\mathfrak{A}$. In this approach the quantisation problem is reduced to description of two-sided ideals $\mathfrak{J} \subset \mathfrak{A}$ which define the commutation relations (the *quantisation ideals*) in the quotient algebras $\mathfrak{A}_\mathfrak{J} = \mathfrak{A}/\mathfrak{J}$ and which are invariant with respect to the dynamics of the system. Surprisingly this idea works rather efficiently and in a number of cases I have been able to quantise the system, i.e. to find commutation relations consistent with the dynamics of the system. A Poisson structure is not assumed in this apprach, but if the Poisson strcture is known, it helps to find quantisation ideals corresponding its deformation.

To illustrate this approach I'll consider the quantisation problem for ODEs on free associative algebra, including N–th Novikov equations and corresponding finite KdV hierarchy. If time permits, I'll discuss quantisation of the Bogoyavlensky family of integrable N–chains (the case $N = 1$ it is a well known Volterra chain):

$$\frac{du_n}{dt} = \sum_{k=1}^{N}(u_{n+k}u_n - u_n u_{n-k}), \qquad n \in \mathbb{Z}$$

and their symmetries. In particular, I will show that odd degree symmetries of the Volterra chain admit two quantisations, and one of them is not deformational.

The talk is based on:

AVM, *Quantisation ideals of nonabelian integrable systems*, arXiv:2009.01838, 2020 (Published in Russ. Math. Surv. v.75:5, pp 199-200, 2020)

V.M.Buchstaber and AVM, *KdV hierarchies and quantum Novikov's equations*, arXiv:2109.06357v1, 2021.

Aleksander Orlov
(Shirshov Institute of Oceanology of RAS)

New integrable matrix models

Using the vertex operator technique we present new class of matrix models. These models are related to the BKP hierarchy introduced by Kyoto school.

Vadim Prokofev
(Skoltech)

Elliptic solutions of Integrable hierarchies of nonlinear differential equations

We consider elliptic solutions of the integrable hierarchies (KP, 2d Toda chain and Matrix KP). It is known that their poles as functions of fist non-trivial time move as particles of classical elliptic integrable models (Calogero-Moser, Ruijsenaars-Schneider and spin Calogero-Moser). We extend these correspondences to the level of hierarchies and show that generating functions of higher Hamiltonians H_k, which govern the dynamics of poles with respect to the k-th hierarchical time, are connected with spectral curves of corresponding systems. We will focus mostly on KP case with mentioning similar results for more general 2d Toda chain and matrix KP cases. The talk is based on works arXiv:2102.03784, arXiv:2103.00214, arXiv:2103.07357

Vladimir Roubtsov

(LAREMA, Département de Mathématiques, ITEP and IITP)

Quantizations of decorated confluent data of Painlevé systems and degenerations of Del Pezzo surfaces

We compare different approaches to quantizing wild character varieties representing monodromy and Stokes data of Painleve's confluent equations. It is shown that the quantum algebras obtained by these approaches coincide. The report is based on the results of joint work with L. Chekhov and M. Mazzokoo ("Adv. Math"., 2021)

Paolo Maria Santini

(Dept. of Physics, University of Roma "La Sapienza")

Towards an analytic description of periodic anomalous waves in nature via the focusing NLS model (joint work with P. G. Grinevich)

The focusing NLS equation is the simplest universal model describing the modulation instability (MI) of quasi monochromatic waves in weakly nonlinear media, and MI is the main physical mechanism for the appearence of anomalous (rogue) waves (AWs) in nature. We first show how the finite gap method adapts to the NLS Cauchy problem for a generic periodic initial perturbation of the unstable background solution, in the case of a finite number N of unstable modes, allowing one to construct the solution, to leading order, in terms of elementary functions of the initial data. In particular, if N=1, one obtains the analytic quantitative description of an exact Fermi-Pasta-Ulam recurrence of AWs already observed in real (water wave and nonlinear optics) and numerical experiments. Then we present the analytic description of the O(1) effect of a small loss or gain on the dynamics of periodic AWs, in full agreements with recent water wave and numerical experiments. At last we discuss analogies and differences with AWs in other contexts: in the Ablowitz-Ladik lattice and in the relativistic field theory described by the massive Thirring model (with F. Coppini).

Ivan Sechin
(Skoltech, Steklov Mathematical Institute of RAS)

Quantum R-matrix identities and Interacting Integrable Tops

Integrability of classical integrable systems, for example, multi-particle Calogero–Moser system, is based on some functional identities on rational, trigonometric, or elliptic functions, which ensure the existence of Lax pair and the Poisson commutativity of integrals of motion. It appears that some quantum R-matrices satisfy the matrix analogues of the relations, known as associative Yang–Baxter equation and its degenerations. This fact allows us to use such quantum R-matrices in Lax pairs instead of scalar functions and construct new classical integrable systems.

I will describe the example of the application of quantum R-matrices relations in classical integrability, introducing the system of interacting integrable tops, generalizing both Calogero systems of particles and Euler tops. I will also show how the resulting integrable structures simultaneously contain the properties of particle and top systems. Relativistic and quantum versions of these systems will be also discussed, if time permits.

Armen Sergeev
(Steklov Mathematical Institute of RAS)

Ginzburg–Landau vortices and Seiberg–Witten equations

Landau-Ginzburg vortices are static solutions of Landau-Ginzburg equations arising in the theory of superconductivity. The moduli space of solutions is described by Taubes' theorem. Slowly moving solutions of dynamical Landau-Ginzburg equations could be investigated in the adiabatic limit. In this limit Landau-Ginzburg equations boil down to adiabatic equations coinciding with Euler equations for geodesics on the moduli space of vortices with respect to the Riemannian metric determined by the kinetic energy of the model.

The analogous procedure of the adiabatic limit could be conducted for the approximate description of solutions of Seiberg-Witten equations on 4-dimensional symplectic manifolds. In this case the geodesics of kinetic metric are substituted

by pseudo holomorphic curves and solutions of Seiberg-Witten equations come down to the family of vortices in planes which are normal to the limiting pseudo holomorphic curve. These families satisfy the non-linear $\bar{\partial}$-equation which could be viewed as a complex analogue of adiabatic equation. Thereafter, the limiting pseudo holomorphic curves could be considered as complex analogues of adiabatic geodesics for Landau-Ginzburg equations in dimenstion $(2+1)$. In this sense Seiberg-Witten model is a $(2+2)$-dimensional analogue of Landau-Ginzburg model.

Alexei Sleptsov
(ITEP)

Quantum R-matrix, quantum 6j-symbols and eigenvalue conjecture

I will talk about the eigenvalue hypothesis, which states that if two quantum R-matrices, being in different irreducible finite-dimensional representations $U_q(sl_N)$, have the same eigenvalues, then the corresponding quantum 6j-symbols also coincide. This hypothesis can be viewed as a new type of symmetry for 6j-symbols. In the case $N = 2$, we obtain the well-known tetrahedron and Regge symmetries, while for $N > 2$, we obtain completely new and previously unknown symmetries, which should be valid in the case of multiplicities as well. Applied to knot theory, these symmetries induce new symmetries of the colored HOMFLY-PT polynomials.

Vladimir Sokolov
(Landau Institute for Theoretical Physics of RAS)

Non-Abelian Poisson brackets on projective spaces

We discuss nonabelian Poisson structures on affine and projective spaces over $\mathbb{C}$. We also construct a class of examples of nonabelian Poisson structures on $\mathbb{C}P^{n-1}$ for $n > 2$. These nonabelian Poisson structures depend on a modular parameter $\tau \in \mathbb{C}$ and an additional descrete parameter $k \in \mathbb{Z}$, where $1 \leq k < n$ and k, n are coprime. The abelianization of these Poisson structures can be lifted to the quadratic elliptic Poisson algebras $q_{n,k}(\tau)$

Vladimir Stukopin
(Moscow Institute of Physics and Technology, MCCME)

Structures of Hopf superalgebras on quantum superalgebras and the quantum Weyl groupoid

Leon Takhtajan
(Stony Brook University, USA, and Euler International Institute, SPb, Russia)

Supersymmetric localization on Lie groups

I will present a new approach to supersymmetric localization that allows to evaluate not only supertraces, but traces of the Euclidean evolution operators. It works for semi-simple compact Lie groups and homogeneous spaces, and I will illustrate it on the example of $U(1)$. This is a joint work with Changha Choi from C.N. Yang Institute for Theoretical Physics.

Sergei Talalov
(Togliatti State University)

"Thirring × Liouville" model and boson–fermion correspondence: the application to string-like dynamical systems

This report is devoted to the application to certain string-like 2D dynamical systems of some models, ideas, methods in 1+1 field theory, proposed by Professor A. K. Pogrebkov at different times.

Alexander Veselov
(Loughborough University, UK, Moscow State University and Steklov Mathematical Institute of RAS, Russia)

Integrable scattering

I will discuss the integrability of the classical systems with scattering, using the explicit example of geodesics on hyperboloids.

Raffaele Vitolo
(Università del Salento)

WDVV equations and invariant bi-Hamiltonian formalism

The WDVV equations are central in Topological Field Theory and Integrable Systems. We prove that in low dimensions the WDVV equations are bi-Hamiltonian. The invariance of the bi-Hamiltonian formalism is proved for N = 3. More examples in higher dimensions show that the result might hold in general. The invariance group of the bi-Hamiltonian pairs is the group of projective reciprocal transformations. The significance of projective invariance of WDVV equations is discussed. Based on a joint work with J. Vasicek, https://arxiv.org/abs/2104.13206

Anton Zabrodin
(Skoltech, NRU HSE, ITEP)

Field analog of the Ruijsenaars-Schneider model from elliptic families of solutions to the 2D Toda lattice

Vladimir Zakharov
(Landau Institute for Theoretical Physics of RAS)

Integrable multidimensional generalizations of the chiral field model