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ABSTRACTS

Monday, December 20

Morning session

Local and global combinatorial structure of torus actions

Anton Ayzenberg

Consider a smooth torus action on a connected closed manifold X of real dimension $2n$ such that the fixed point set is finite and nonempty. The action is called equivariantly formal if odd degree cohomology of X vanishes. For example, hamiltonian actions on compact manifolds with isolated fixed points are equivariantly formal.

Connected components of X^G, for some subgroup G of T, are called face submanifolds of X. We study the graded poset S_X of face submanifolds. The motivating and well studied examples are actions of comlexity zero (torus manifolds). In these examples S_X (with reversed order) is a simplicial poset - this is a general local property. If X is equivariantly formal, S_X is a triangulated sphere - this is a global property.

In my talk I want to show how these facts generalize to positive complexity. The talk is based on several works (as well as works in progress) with Mikiya Masuda, Grigory Solomadin and Vladislav Cherepanov.

Cohomological rigidity of manifolds associated to ideal right-angled hyperbolic 3-polytopes

Nikolai Erokhovets

Toric topology assigns to each n-dimensional combinatorial simple convex polytope P with m facets an $(m+n)$-dimensional moment-angle manifold $\mathcal{Z}_P$ with an action of a compact torus T^m such that $\mathcal{Z}_P/T^m$ is a convex polytope of combinatorial type P.

Definition 1. A simple n-polytope is called *B-rigid*, if any isomorphism of graded rings $H^*(\mathcal{Z}_P,\mathbb{Z}) = H^*(\mathcal{Z}_Q,\mathbb{Z})$ for a simple n-polytope Q implies that P and Q are combinatorially equivalent.

An *ideal almost Pogorelov polytope* is a combinatorial 3-polytope obtained by cutting off all the ideal vertices of an ideal right-angled 3-polytope in the Lobachevsky (hyperbolic) space $\mathbb{L}^3$. These polytopes are exactly the polytopes obtained from any, not necessarily simple, convex 3-polytopes by cutting off all the vertices followed by cutting off all the "old" edges. The boundary of the dual polytope is the barycentric subdivision of the boundary of the old polytope (and also of its dual polytope).

Theorem. Any ideal almost Pogorelov polytope is B-rigid.

Definition 2. A family of manifolds is called *cohomologically rigid* over the ring R, if for any two manifolds M and N from the family any isomorphism of graded rings $H^*(M,R) \simeq H^*(N,R)$ implies that M and N are diffeomorphic.

Any ideal almost Pogorelov polytope P has a canonical colouring of facets in 3 colours corresponding to vertices, edges and facets of the polytope that gives P via cutting off vertices and "old" edges. This colouring produces the 6-dimensional quasitoric manifold $M(P)$ and the 3-dimensional small cover $N(P)$, which are known as "pullbacks from the linear model".

Corollary. The families $\{\mathcal{Z}_P\}$, $\{M(P)\}$, and $\{N(P)\}$ indexed by the ideal right-angled hyperpolic 3-polytopes are cohomologically rigid over $\mathbb{Z}$, $\mathbb{Z}$ and $\mathbb{Z}_2$ respectively.

We also plan to discuss the geometry of the 3-manifolds $N(P)$.

References

[E20] N. Erokhovets, *B-rigidity of ideal almost Pogorelov polytopes*, arXiv:2005.07665v3.

[E20] Nikolai Yu. Erokhovets, *B-rigidity of the property to be an almost Pogorelov polytope*, Contemporary Mathematics, 772, 2021, 107–122.

Orbit space $G_{n,2}/T^n$ and Chow quotient $G_{n,2}//(\mathbb{C}^*)^n$

Svjetlana Terzić

The complex Grassmann manifolds $G_{n,2}$ are of special interest as they have several remarkable properties which distinguish them from $G_{n,k}$ for $k > 2$. In this talk we present an explicit construction of the model $U_n = \Delta_{n,2} \times \mathcal{F}_n$ for the orbit space $G_{n,2}/T^n$ in a sense that there exists a continuous surjection $U_n \to G_{n,2}/T^n$, where $\Delta_{n,2}$ is a hypersimplex and $\mathcal{F}_n$ is a smooth, compact manifold. In addition, we provide an explicit description of $\mathcal{F}_n$ by the method of wonderful compactification and prove that it coincides with Grothendieck-Knudsen compactification $\overline{\mathcal{M}(0,n)}$ of n-pointed curves of genus zero, that is with the Chow quotient $G_{n,2}//(\mathbb{C}^*)^n$. As a corollary we describe the build up points in this compactification in terms of the ingredients for the model U_n.

The talk is based on joint results with Victor M. Buchstaber.

Evening session

Multi-polytopes and the BKK theorem for quasitoric bundles

Leonid Monin

Quasitoric bundles are one of the easiest objects in parametric toric topology. These are locally trivial fiber bundles with a quasitoric manifold as fiber. In my talk I will discuss the connection between quasitoric bundles and geometry of polytopes. First I will talk about the theory of multi-polytopes introduced by Masuda and their connection to quasitoric manifolds. Then I will discuss a class of smooth polynomial measures on multi-polytopes. I will finish the talk with a generalisation of the classical BKK theorem to the case of quasitoric bundles. This result connects the intersection numbers in cohomology rings of quasitoric bundles to the geometry of multi-polytopes. The talk is based on joint works with Askold Khovanskii and Ivan Limonchenko.

Musing over algebraic models for the cohomology of moment-angle complexes

Jelena Grbić

Reporting on an ongoing joint work with Martin Bendersky.

Describing the cohomology of (real and complex) moment-angle complexes (in their many guises) has been a motivating problem that has driven research developments in a host of mathematical areas; commutative algebra, combinatorics, configuration spaces, algebraic geometry, toric topology, just to name a few.

In the talk I shall recall some of algebraic models and illustrate that, although seemingly different, they have the same direct interpretation in geometry of constitutional topological spaces and combinatorics of simplicial complexes.

Tuesday, December 21

Morning session

Toric topology and persistent homology

Ivan Limonchenko

Based on the results of Baskakov, Buchstaber, Panov, and Franz, one possible way to apply toric topology in topological data analysis would be to define and study the properties of bigraded persistent (co)homology and bigraded barcode of a data set using (co)homology of (real) moment-angle-complexes over the corresponding Vietoris-Rips complexes.

However, as we will show in the talk, this version of a bigraded barcode satisfies only a weaker version of the stability theorem. Applying the so called double cohomology of a moment-angle-complex instead of the ordinary one, we will fix this problem and introduce such a construction of the bigraded persistent (co)homology that the corresponding bigraded barcode satisfies the stability theorem.

Based on the ongoing research project joint with A.Bahri, T.E.Panov, J.Song, and D.Stanley.

Pontryagin algebras and the LS-category of moment-angle complexes in the flag case

Fedor Vylegzhanin

We study the Pontryagin algebras (the loop homology) of moment-angle complexes corresponding to flag simplicial complexes. The Poincare series, an explicit minimal set of generators and full description of free and one-relator such algebras are known due to the works of Panov-Ray, Grbic-Panov-Theriault-Wu and Grbic-Ilyasova-Panov-Simmons.

We introduce the natural multigrading and describe the degrees of minimal relations and multigraded Poincare series. We prove the collapse of the Milnor-Moore spectral sequence, and use this to calculate the Lusternik-Schnirelmann category of such moment-angle complexes.

On the top homology of the Torelli group and the Johnson kernel

Igor Spiridonov

Mapping class groups of oriented surfaces are closely related to moduli spaces of complex curves, and to topology of 3-manifolds. The "non-linear part" of the mapping class group is the Torelli subgroup $\mathcal{I}_g$, consisting of all mapping classes acting trivially on the first homology of the surface. From a topological point of view, this subgroup is interesting because of its connection to homology 3-spheres. The Torelli group also arises in algebraic geometry as the fundamental group of the Torelli space, the moduli space of smooth complex curves with homology framings. The first homology group $\mathrm{H}_1(\mathcal{I}_g, \mathbb{Z})$ was described by Johnson in the 1980's. However, none of the other non-zero homology groups has been computed explicitly. In this talk, we discuss the problem of computing the homology of the Torelli group and the Johnson kernel $\mathcal{K}_g$, which is the most well-studied subgroup of $\mathcal{I}_g$. We give a complete description of the Torelli group top homology group in genus 3. Also, we describe the subgroup of the Johnson kernel top homology group, generated by abelian cycles determined by Dehn twists about disjoint separating curves. The main approach is the study of the action of $\mathcal{I}_g$ and $\mathcal{K}_g$ on the complex of cycles, introduced by Bestvina, Bux and Margalit in 2007.

Evening session

Iterated higher Whitehead products and Adams–Hilton models for polyhedral products

Elizaveta Zhuravleva

In this talk I discuss higher Whitehead products, invariants in unstable homotopy theory, which are considered in the context of the studying Davis—Januszkiewicz spaces and moment-angle complexes.

It is known that rational homotopy groups of loop space form the homotopy Lie algebra in which the Jacobi identity holds. There is a structure of L_∞ algebra, the generalization of Lie algebra for which we have n-ary brackets that satisfy the generalized Jacobi identities. In general, we do not know, what relations hold for (canonical) higher Whitehead products.

In this talk I introduce an algebraic construction that gives us chains in the cobar–construction of the homology of Davis—Januszkiewicz space representing Hurewicz images of higher iterated Whitehead products. For this purpose we exhibit Adams–Hilton models for Davis—Januszkiewicz spaces and polyhedral product of spheres (for arbitrary simplicial complexes). Using these chains one can obtain relations on (canonical) higher Whitehead products.

There is a minimal simplicial complex $\mathcal{K} = \partial\Delta(\partial\Delta(1,2,3),4,5)$, for which iterated Whitehead products $[[\mu_1,\mu_2,\mu_3],\mu_4,\mu_5]$ is defined. In this talk I represent the full description of Pontryagin algebra and homotopy Lie algebra of Davis—Januszkiewicz space for this $\mathcal{K}$, using Whitehead products. We will see that relations on Whitehead products have the form of L-infinity identities.

The Lie algebra associated with the lower central series of a right-angled Coxeter group

Yakov Veryovkin

We study the lower central series of a right-angled Coxeter group $RC_{\mathcal{K}}$ and the associated Lie algebra $L(RC_{\mathcal{K}})$. The latter is related to the graph Lie algebra $L_{\mathcal{K}}$. We give an explicit combinatorial description of the first three consecutive factors of the lower central series of the group $RC_{\mathcal{K}}$.