International Conference

"Categories and Birational Geometry"

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Organizers

Steklov Mathematical Institute of Russian Academy of Sciences, Moscow

Steklov International Mathematical Center, Moscow

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ABSTRACTS

Nicolas Addington

Title: Hodge numbers are not derived invariants in positive characteristic

Abstract: Over the complex numbers, the Hodge numbers of a 3-fold are invariants of the derived category, but I will show that this fails over $\overline{F_3}$. The counterexample is a pair of 3-folds fibered over P^1 in Abelian surfaces, similar to the example of Gross, Popescu, Bak, and Schnell which shows that the fundamental group and Brauer group are not derived invariants. There are also connections with special cubic fourfolds of discriminant 18. This is joint work with Dan Bragg.

Federico Barbacovi

Title: The flop-flop autoequivalence via spherical functors

Abstract: The derived category of a scheme X is an algebraic object whose interest in geometry is beyond doubt. Among the various things one can be interested in about $D(X)$, there are its symmetries, i.e its autoequivalences. The study of the autoequivalences of $D(X)$ has attracted a lot of attention, and although in some cases this group is completely described, there is no "general" recipe to construct all the autoequivalences of $D(X)$. A (conjectural) source of autoequivalences are flops. In this talk, I will discuss the so called "flop-flop" autoequivalence, and explain how it can be realised as the (inverse) spherical twist around a spherical functor.

Agnieszka Bodzenta

Title: Reconstruction of a surface from the category of reflexive sheaves

Abstract: I discuss the category $\mathrm{Ref}(X)$ of reflexive sheaves on a normal Cohen-Macaulay surface X. I describe the canonical structure of an exact category on $\mathrm{Ref}(X)$. I introduce the notion of the right abelian envelope of an exact category and use it to reconstruct a surface proper over an affine scheme from the category of reflexive sheaves on it. It is a joint work in progress with A. Bondal.

John Calabrese

Title: Gabriel's reconstruction theorem and birational geometry

Abstract: A famous theorem of Gabriel states that two varieties are isomorphic if and only if their categories of coherent sheaves are equivalent. On the other hand, two varieties are birational if and only if their function fields are isomorphic. In this talk I will discuss an elementary result which interpolates between these two theorems. Namely, taking the quotient of $\mathrm{Coh}(X)$ by sheaves supported in codimension c (or bigger) controls the isomorphism type of X in codimension c. This is joint work with Roberto Pirisi.

Will Donovan

Title: Classification of simple flops and variation of GIT

Abstract: Though classification of flops remains very challenging in general, progress on classification of simple flops has been made by D. Li and A. Kanemitsu, focusing on their relation with Fano manifolds. Derived equivalence are conjectured for all, and remain open in many cases. I review this, and discuss approaches to proving new equivalences using variation of GIT, in joint work with Weilin Su.

Zheng Hua

Title: Noncommutative Mather-Yau theorem

Abstract: I will survey several Mather-Yau type theorems in noncommutative differential calculus. They can be viewed as implicit function theorems for functions with singularities in noncommutative geometry. In the categorical approach of birational geometry, noncommutative Mather-Yau theorem can be used to lift certain isomorphism of cohomology algebra to the chain level. This is joint work with Gui-song Zhou and with Bernhard Keller.

Martin Kalck

Title: Semiorthogonal decompositions on singular varieties

Abstract: Inspired by recent work of Kawamata, we introduce and study certain semiorthogonal decompositions on singular projective varieties, which generalize full ex-ceptional sequences for smooth varieties.

We will first present constructions of such decompositions. Using singularity categories, negative K-theory, NCCRs and cluster theory, we will then discuss obstructions to the ex-istence of such decompositions. This is joint work with N. Pavic and E. Shinder.

Alexander Kuznetsov

Title: Simultaneous categorical resolutions of singularities

Abstract: It is a classical fact, that for a family of surfaces over a smooth curve with smooth general fiber, if the special fiber has only rational double points as singularities, then after a possible finite base change, one can resolve the singularities of the total space and the special fiber by a common blowup. I will talk about a categorical version of this construction that also works in higher dimensions and about its applications.

Amnon Neeman

Title: Uniqueness theorems for dg enhancements

Abstract: For more than twenty years there has been much interest in results describing which triangulated categories have unique enhancements, and results giving conditions sufficient to guarantee that certain functors must be Fourier-Mukai. We will begin the talk with a brief survey of this literature.

Then we will move to recent work, joint with Alberto Canonaco and Paolo Stellari, which substantially improves on what was known about uniqueness of enhancements. After stating the main results we will sketch the key new ideas that go into the proofs.

Shinnosuke Okawa

Title: Moduli space of semiorthogonal decompositions

Abstract: For smooth projective families of algebraic varieties we introduce certain moduli spaces which, roughly speaking, classify the semiorthogonal decompositions of the derived category of the fibers. We explain their basic properties and a few applications. Comparison to families of extremal contractions will be also discussed.

Evgeny Shinder

Title: Derived categories of Fano threefolds of index two with ordinary double points

Abstract: A Kawamata decomposition of the derived category of a singular variety is a semiorthogonal decomposition into a perfect part and derived categories of finite-dimensional algebras. Previously known examples of singular threefolds admitting Kawamata decompositions are nodal V_6 and a nodal quadric. In this talk I will explain that a necessary and sufficient condition for a nodal Fano threefold of index two to admit a Kawamata decomposition is maximal nonfactoriality, which means that Weil divisors separate the singular points. This is joint work in progress with Nebojsa Pavic (Hannover).

Yukinobu Toda

Title: Categorified Hall products for local surfaces and wall-crossing

Abstract: The two dimensional categorified Hall algebra is introduced by Porta-Sala as a categorification of cohomological Hall algebra for surfaces by Kapranov-Vasserot. In this talk I will show that categorified Hall products induce actions to Donaldson-Thomas categories for local surfaces. Then I propose the existence of semi-orthogonal decompositions of DT categories under wall-crossing described by categorified Hall prod-ucts, which in particular implies the d-critical analogue of D/K conjecture. I will focus on the example of MNOP/PT wall-crossing, and prove the conjecture for reduced curve classes.

Michael Wemyss

Title: K(pi,1) via contraction algebras

Abstract: Usually in algebraic geometry settings, such as for a flopping contraction $X \to \operatorname{Spec} R$, we are interested in establishing contractibility of the stability manifold. Actually, this is not strictly true: we are usually interested in proving contractibility of certain components of the stability manifold of certain subcategories of $\mathrm{Db}(\operatorname{coh} X)$. Depending on which components, and on which subcategories, the difficulty of the problem varies.

I will explain how to approach the contractibility in one such case, by "mirroring" the stability manifold from the image of a spherical functor, to the source category of the spherical functor. It turns out that contractibility is much easier there. There are three main corollaries: K(pi,1) for all intersection arrangements in ADE root systems (which includes the Coxeter groups I_n with $n = 3, 4, 5, 6, 8$), plus faithfulness of group actions in various settings (the first avoiding normal forms), plus contractibility of stability manifolds in some 3-CY settings. If there is time, I will explain some of the issues in the "infinite" case. This is joint work with Jenny August.