Seventh Workshop on Combinatorics of Moduli Spaces Cluster Algebras and Topological Recursion

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Organizers

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Joergen Ellegaard Andersen
(Center for Quantum Mathematics, Danish Institute for Advanced Study,
Syddansk Universitet, Odense, Denmark)

Geometric Recursion

We shall review the geometric recursion and its relation to topological recursion. In particular, we shall consider the target theory of continuous functions on Teichmüller spaces and we shall exhibit a number of classes of mapping class group invariant functions, which satisfies the geometric recursion. Many of these classes of functions are integrable over moduli spaces and we prove that these averages over moduli spaces satisfies topological recursion. The talk will end with a discussion of open Geometric Recursion. The talk is based on joint work with Borot and Orantin.

Marco Bertola
(Concordia University, Montreal, Canada, SISSA, Trieste, Italy)

Graph Connections, (Wild) Character Varieties, and Generating Functions in Symplectic Geometry

We will discuss a natural (pre)-symplectic structure associated to an arbitrary flat graph connection on a Riemann surface and its invariance properties.

This allows to efficiently parametrize (wild) character varieties using Fock-Goncharov coordinates and provide explicit log-canonical coordinates for several types of Poisson structures; Goldman on the standard character variety, Flaschka-Newell-Boalch on Stokes' manifolds and Ugaglia-Bondal Poisson structures.

In the case of (wild) character varieties, this construction allows to define the generating functions of symplectic polarizations and identify them with the classical notion of isomonodromic tau functions of the Japanese school.

Based on works with Dmitry Korotkin, Fabrizio Del Monte, Sofia Tarricone.

Gaetan Borot
(Humboldt University, Berlin, Germany)

$N = 2$ Supersymmetric Gauge Theories and Topological Recursion

I will explain how Whittaker vectors for principal W-algebras can be computed by topological recursion, using the formalism of Airy structures. As a result, we can access the Nekrasov partition function of $N = 2$ pure gauge theory on S^4 in general Omega-

background. This is a joint work with Vincent Bouchard, Nitin Kumar Chidambaram and Thomas Creutzig.

Andrea Brini
(Sheffield University, UK)

Stable Maps to Looijenga Pairs

A Looijenga pair is a pair (X, D) with X a smooth complex projective surface and D a singular anticanonical divisor in X. I will describe a series of correspondences relating five different classes of string-theory motivated invariants specified by the geometry of (X, D):

- the log Gromov--Witten theory of (X, D),
- the Gromov--Witten theory of X twisted by the sum of the dual line bundles to the irreducible components of D,
- the open Gromov--Witten theory of special Lagrangians in a toric Calabi--Yau 3-fold determined by (X, D),
- the Donaldson--Thomas theory of a symmetric quiver specified by (X, D), and
- a class of BPS invariants considered in different contexts by Klemm--Pandharipande, Ionel--Parker, and Labastida--Marino--Ooguri--Vafa.

I will also show how the problem of computing all these invariants is closed-form solvable. Joint work with P. Bousseau and M. van Garrel.

Boris Bychkov
(HSE University, Moscow, Russia)

Topological Recursion for Hypergeometric KP/Generalized Hurwitz n-Point Functions

Olivia Dumitrescu
(University of North Carolina, Chapel Hill, USA)

Lagrangian Correspondence Between Hitchin and De Rham Moduli Spaces

In 2008 Simpson conjectures the existence of a holomorphic Lagrangian foliation in the de Rham moduli space of holomorphic G-connections for a complex reductive group G. I will present an algebraic geometry description of the Lagrangian correspondence of conformal limits, based on the work of Simpson for $SL_2(C)$-Higgs bundles defined on a smooth connected projective curve C of genus at least 2.

This talk is based on joint work with Jennifer Brown and Motohico Mulase.

Petr Dunin-Barkovskii
(HSE University, Moscow;
Skolkovo Institute of Science and Technology, Russia)

Explicit Closed Algebraic Formulas for Hypergeometric KP/Generalized Hurwitz n-Point Functions

Elba Garcia-Failde
(Universite de Paris, France)

Generalised Kontsevich Graphs, r-Spin Intersection Numbers and Topological Recursion

In 1990, Witten formulated his celebrated conjecture that predicts that the generating series of intersection numbers of psi-classes is a tau function of the KdV hierarchy. Kontsevich gave the first proof of this conjecture making use of a cell decomposition of a combinatorial model of the moduli space of curves by means of certain ribbon graphs which are Feynman graphs of a cubic hermitian matrix model with an external field. Together with Raphaël Belliard, Séverin Charbonnier and Bertrand Eynard, we studied certain generalisations of these graphs and showed that they satisfy a Tutte recursion. This implies a combinatorial interpretation of universal expressions that we transform into loop equations for a large class of spectral curves. I will show how we arrived to the topological recursion statement for this model and how we related a particular instance of it to intersection numbers with Witten's r-spin class, deducing also that r-spin intersection numbers can be computed by higher topological recursion. I will finish with comments on further consequences of our work that we would like to explore in the future.

Ilia Gaiur

(University of Birmingham, UK)

Irregular Isomonodromic Deformations: Hamitlonian Theory and Quantization

This talk is dedicated to the isomonodromic deformation equations on the Riemann sphere with the punctures of an arbitrary Poincaré rank (regular and irregular isomonodromic problems). Such deformations are closely related to the Painlevé equations and Garnier systems, as well as to the moduli space of flat connections over the Riemann sphere with boundaries. In this talk, using the confluence procedure I will show what Poisson and symplectic structure arise when we transfer to the irregular case. Moreover, I am going to discuss how the connection depends on irregular deformation parameters and their meaning from the representation theory point of view. I am also

going to discuss the quantization of obtained isomonodromic systems and relation to the quasi-classical solutions of the KZ equations written in the terms of the classical isomonodromic tau function

Alessandro Giacchetto
(Max Planck Inst. for Math., Bonn, Germany)

Multicurve Count, Masur–Veech Volumes and Topological Recursion

Based on the work of Mirzakhani, I will explain how the count of multicurves on bordered surfaces can be solved by geometric and topological recursion techniques, both in the hyperbolic and in the combinatorial setting. I will also explain how the asymptotics of such counts compute the Masur–Veech volumes of the moduli space of quadratic differentials. The talk is based on joint works with J. E. Andersen, G. Borot, S. Charbonnier, V. Delecroix, D. Lewański and C. Wheeler.

Kohei Iwaki
(University of Tokyo, Japan)

Topological recursion, Painlevé Tau-Function and Exact WKB Analysis

I'll show that the solution of the first Painlevé equation (together with the isomonodromic wave function which satisfies the associated linear system) are constructed as the discrete Fourier transform of the topological recursion partition function. Assuming certain conjectures on a resurgence property of the formal series constructed from topological recursion, I'll also explain how the Stokes multipliers of associated linear system are computed from the view point of exact WKB analysis.

Omar Kidwai
(University of Tokyo, Japan)

Topological Recursion and Uncoupled BPS Structures for Hypergeometric Spectral Curves

The notion of BPS structure formalizes many of the structures appearing in the study of four-dimensional $N = 2$ QFTs by Gaiotto-Moore-Neitzke as well as Bridgeland's spaces of stability conditions and the generalized Donaldson-Thomas (equivalently, BPS) invariants. We outline a correspondence which relates the BPS invariants, central charges, and solutions to certain Riemann-Hilbert problems with the topological recursion free energies and Voros symbols of corresponding quantum curves, which we have shown for the special case of spectral curves of "hypergeometric type". This is joint work with K. Iwaki, arXiv:2010.05596 + ongoing.

Danilo Lewanski
(Université Paris-Saclay, Institut de Physique Théorique,
Institut des Hautes Études Scientifiques, France)

A New Spin for Hurwitz Numbers and Chiodo Classes

Standard Hurwitz theory provides a large class of examples for topological recursion and generally obeys 2D Toda integrability. The Gromov-Witten / Hurwitz correspondence links it to the target P^1, and its corresponding cohomology representation on the moduli spaces of curves recovers classes studied independently.

The ‘spin' counterpart of the story recently received some attention, especially from the integrability side where it is linked to the BKP hierarchy, and from the Gromov-Witten-Witten side where it is connected to Kähler surface targets. We investigate how topological recursion behaves for this problem, encountering quotients of equivariant spectral curves. Moreover, we apply Givental formalism to compute the corresponding cohomology class, where Witten’s CohFT makes its appearance. Based on a recent work with A.Giacchetto and R. Kramer.

Paul Norbury
(Melbourne University, Australia)

Volume of the Moduli Space of Super Riemann Surfaces

I will describe a relationship between different approaches to calculating the volume of the moduli space of super hyperbolic surfaces. Stanford and Witten defined the volume of the moduli space of super hyperbolic surfaces via a natural measure on the character variety of a surface into a supergroup. Alternatively, the volume of a symplectic supermanifold can be quite generally expressed in terms of the Euler class of a bundle over an underlying symplectic manifold together with its symplectic form. These two viewpoints lead to a conjectural relationship between a collection of integrals of known cohomology classes over the moduli space of stable Riemann surfaces and the volume of the moduli space of super Riemann surfaces.

Kento Osuga
(Sheffield University, UK)

$N = 1$ Super Topological Recursion

The notion of the $N = 1$ super topological recursion will be proposed, which can be thought of as a supersymmetric generalisation of the well-known topological

recursion. An equivalent description in terms of super Airy structures will also be given, and I will show a dictionary between them. I will then discuss a few applications of the $N = 1$ super topological recursion to 2d supergravity. This is a joint work with Vincent Bouchard.

Misha Shapiro
(Michigan State University, East Lansing, USA)

Darboux Coordinates for Symplectic Groupoid

We explain construction of Darboux coordinates for generic symplectic leaves of the groupoid of unipotent upper triangular matrices using Fock-Goncharov coordinates for framed moduli space of flat connections and express the braid group action as sequence of cluster mutations. This is a joint project with L. Chekhov.

Linhui Shen
(Michigan State University, East Lansing, USA)

Moduli Spaces of G-Local Systems and Poisson Feometry

Let G be a split semi-simple algebraic group over Q. We introduce a natural cluster Poisson structure on moduli spaces of framed G-local systems over surfaces with marked points. As a consequence, the moduli spaces of G-local systems admit natural Poisson structures, and can be further quantized. We will study the principal series representations of such quantum spaces. If time permits, I will discuss its applications in the study of quantum groups. This talk will mainly be based on joint work with A. B. Goncharov (arXiv:1904.10491).

Karoline van Gemst
(Sheffield University, UK)

Mirror Symmetry for Dubrovin-Zhang Frobenius Manifolds

A Frobenius manifold is a complex manifold with a certain algebraic structure on its tangent bundle, and some additional axioms. They arise in various different contexts in mathematics, with the main sources being quantum cohomology, singularity theory and Lie theory. In 1998, Boris Dubrovin and Youjin Zhang constructed Frobenius structures on the orbit space of a certain extension of affine Weyl groups associated to simple complex Lie algebras. In this talk I will, after introducing the concept of

Frobenius manifolds, explain what mirror symmetry means in this context, and present the results of a recent preprint where we obtain B-model mirrors for all Dubrovin-Zhang manifolds defined in 1998. If time permits, I will also mention an interesting application of this work. This is joint work with Andrea Brini and based on arXiv: 2103.12673.