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Abstracts

Farit Avkhadiev (*Kazan Federal University*)

Hardy and Rellich type conformally invariant inequalities

For real-valued test functions $u \in C_0^\infty(\Omega)$ we study conformally invariant integral inequalities on domains Ω of the Euclidean space $\mathbb{R}^n$, $n \geq 2$. To construct conformally invariant integrals we use the hyperbolic radius $R = R(x, \Omega)$ defined as follows. If $n = 2$ then $1/R(x, \Omega)$ is the coefficient of the Poincaré metric with Gaussian curvature $\kappa = -4$. If $n = 3$ then the hyperbolic radius is determined by $V = V(x) :\equiv R^{1-n/2}(x, \Omega)$, where V is the maximal solution of the Liouville differential equation $\Delta V = n(n-2)V^{(n+2)/(n-2)}$. In particular, $R(x, B) = 1 - |x|^2$ for the unit ball $|x| < 1$ and $R(x, H^+) = 2x_1$ for the half-space $x_1 > 0$. In the case $n \geq 3$ Loewner and Nirenberg are proved the existence of $R(x, \Omega)$ for domains Ω satisfying certain conditions on the boundary. If there exists $R(x, \Omega)$ we will say that Ω is a domain of hyperbolic type.

By Δu and ∇u we denote the Euclidean Laplacian and the Euclidean gradient of the function u. It is very known that the Dirichlet integral $\int_\Omega |\nabla u(x)|^n dx$ is conformally invariant. We use some generalizations of this fact. Suppose that $p \in [1, \infty)$ is a fixed number. We show that the integrals $\int_\Omega |u|^p R^{-n}(x, \Omega))dx$ and $\int_\Omega |\nabla u|^p R^{p-n}(x, \Omega)dx$ are conformally invariant for domains of dimension $n \geq 2$ and that the integral $\int_\Omega |\Delta u|^p R^{2p-n}(x, \Omega)dx$ is conformally invariant for $n = 2$, but it is invariant with respect to linear conformal mappings, only, in the case $n \geq 3$.

There are several results on the conformally invariant inequalities of Hardy and Rellich type in the case $n = 2$ (see, for instance, Avkhadiev, F.G. "Integral inequalities in domains of hyperbolic type and their applications". Sbornik: Mathematics, 2015, 206(12), 1657–1681). Now we present some results for the case $n \geq 2$ from our paper in press (see Avkhadiev, F.G. "Conformally invariant inequalities on domains of the Euclidean space", to be published in Izvestiya: Mathematics, 2019). By $(\nabla u, \nabla R)$ we will denote the scalar product in $\mathbb{R}^n$.

Theorem 1. *Suppose that $n \geq 3$, $1 \leq p < \infty$, and that $B = \{x \in \mathbb{R}^n : |x| < 1\}$. Then*

$$\int_B \frac{|\nabla u(x)|^p \, dx}{(1 - |x|^2)^{n-p}} \geq \frac{2^p (n-1)^p}{p^p} \int_B \frac{|u(x)|^p dx}{(1 - |x|^2)^n} \quad \forall u \in C_0^\infty(B).$$

The constant $2^p(n-1)^p/p^p$ is sharp, i. e. it is the best possible one at this place.

Theorem 2. *Suppose that $n \geq 3$ and that $\Omega \subset \mathbb{R}^n$ is a domain of hyperbolic type. Then*

$$\int_\Omega \frac{|\nabla u(x)|^2 dx}{R^{n-2}(x, \Omega)} \geq n(n-2) \int_\Omega \frac{|u(x)|^2 dx}{R^n(x, \Omega)} \quad \forall u \in C_0^\infty(\Omega),$$

and

$$\int_\Omega |\nabla u(x)|^n dx \geq 2^n \left(1 - 2/n\right)^{n/2} \int_\Omega \frac{|u(x)|^n dx}{R^n(x, \Omega)} \quad \forall u \in C_0^\infty(\Omega).$$

Theorem 3. *Suppose that $n \geq 2$, $1 \leq p < \infty$, $1 + n/2 \leq s < \infty$, and that $\Omega \subset \mathbb{R}^n$ is a domain of hyperbolic type. Then*

$$\int_\Omega \frac{|(\nabla u(x), \nabla R(x, \Omega))|^p \, dx}{R^{s-p}(x, \Omega)} \geq \frac{2^p n^p}{p^p} \int_\Omega \frac{|u(x)|^p dx}{R^s(x, \Omega)} \quad \forall u \in C_0^\infty(\Omega).$$

If Ω is a half-space and $s = 1 + n/2$, then the constant $2^p n^p/p^p$ in this inequality is sharp.

The integrals from theorem 3 are invariant with respect to linear conformal mappings.

This work was supported by the Russian Science Foundation under Grant no. 18-11-00115.

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Francesco Dell'Accio (*University of Calabria*)

Multinode Rational Operators for Scattered Data Interpolation

In 1968 D. Shepard [1] introduces an approximation method for the interpolation of scattered data which consists in a weighted average of functional values at the data points. The method is easy to implement (indeed it is the fastest method for the interpolation of scattered data [2] but it reproduces exactly only constant polynomials and has flat spots in the neighbourhood of all data points. In 1983 F. Little [3] considers weighted average of local linear interpolants based on triples of data sites and takes as basis functions the normalization of the product of inverse distances from the points of the triples. This method overcomes the drawbacks of the Shepard method and, at the same time, maintains its features of simplicity of implementation and speed. In fact, the use of a searching technique to detect and select the nearest neighbor points [4] to determine the best local linear interpolant on compact triangulations [5], allows to consider the triangular Shepard method a fast meshfree method with an adequate order and a good accuracy of approximation. As Little suggests, his method can be generalized to higher dimensions and to sets of more than three points. In this talk we will discuss about some of these generalizations. (Joint work with Filomena Di Tommaso.)

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Roland Duduchava (*The University of Georgia; A.Razmadze Mathematical Institute, Tbilisi, Georgia*)

Boundary value problems on hypersurfaces and Γ-convergence

We consider two examples of boundary value problems (BVPs) on hypersurfaces: heat conduction by an "isotropic"media, governed by the Laplace equation and bending of elastic "isotropic"media governed by Láme equations. The boundary conditions are classical Dirichlet-Neumann mixed type. The domain $\Omega^h := \mathcal{C}\times(-h,h)$ is of thickness $2h$. Here $\mathcal{C}\subset\mathcal{S}$ is a smooth subsurface of a closed hypersurface $\mathcal{S}$ with smooth nonempty boundary $\partial\mathcal{C}$.

The object of the investigation is what happens with the above mentioned mixed boundary value problems when the thickness of the layer converges to zero $h\to 0$. It is shown that the corresponding BVPs converge in the sense of Γ-convergence to a certain BVPs on the mid surface $\mathcal{C}$: The BVP for the Laplace equation converges to the BVP for the Dirichlet BVP for the Laplace-Beltrami equation, while for the Láme equation we get a new form of BVP for the shell equation.

The suggested approach is based on the fact that the Laplace and Láme operators are represented in terms of Günter's tangential and normal (to the surface) derivatives. Namely, if ν is the unit normal vector field on the surface, extended in the domain Ω_h, the Günter's derivatives read

$$\mathcal{D}_j := \partial_j - \nu_j \mathcal{D}_4, \qquad \mathcal{D}_4 = \partial_\nu = \sum_{k=1}^{3} \nu_k \partial_k, \qquad j = 1, \dots, n$$

and the Laplace-Beltrami operator on the surface $\mathcal{C}$ is represented as follows

$$\Delta_{\mathcal{C}} = \mathcal{D}_1^2 + \mathcal{D}_2^2 + \mathcal{D}_3^2.$$

Moreover, the Laplace and the Láme operators in the domain

$$\Delta_{\Omega^h} = \partial_1^2 + \partial_2^2 + \partial_3^2, \qquad \mathcal{L}_{\Omega^h} = -2\mu\,\Delta - (\lambda + 2\mu)\,\nabla \mathrm{div}$$

are represented as follows:

$$\Delta_{\Omega^h} = \sum_{j=1}^{4} \mathcal{D}_j^2 + 2\mathcal{H}_{\mathcal{C}} \mathcal{D}_4, \qquad \mathcal{L}_{\Omega^h} = -2\mu\,\Delta_{\Omega^h} - (\lambda + 2\mu)\,\nabla_{\Omega^h} \mathrm{div}_{\Omega^h}.$$

Here $\mathcal{H}_{\mathcal{C}}$ is the mean curvature of the surface $\mathcal{C}$ and

$$\nabla_{\Omega^h} \varphi := \Big\{ \mathcal{D}_1 \varphi, ..., \mathcal{D}_4 \varphi \Big\}^\top, \qquad \mathrm{div}_{\Omega^h} \mathbf{U} := \sum_{j=1}^{4} \mathcal{D}_j U_j^0 + \mathcal{H}_{\mathcal{C}} U_4^0,$$
$$\mathbf{U} = (U_1, U_2, U_3)^\top, \qquad U_j^0 := U_j - U_4^0, \quad U_4^0 := \langle \nu, \mathbf{U} \rangle, \quad j = 1, 2, 3$$

are the gradient and divergence.

The work is carried out in collaboration with T. Buchukuri and G. Tephnadze (Tbilisi).

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Pavel Exner (*Doppler Institute for Mathematical Physics and Applied Mathematics, Prague*)

Optimization of the principal eigenvalue in various geometries

Search for shapes which optimize the principle eigenvalue of a PDE problem is a trademark problem of mathematical physics. In this talk we shall address questions of that type for Schrödinger operators with an attractive singular 'potential', supported by a manifold or a geometric complex, which can be formally written as $-\Delta - \alpha\delta(x - \Gamma)$ with $\alpha > 0$. Various classes of the interaction support Γ will be discussed: loops, manifolds homothetic to a sphere, cones and stars; while most attention will be paid to the case $\mathrm{codim}\,\Gamma = 1$, we will consider also situations where the interaction is strongly singular, either of the δ' type or with the support of codimension two.

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Huijun Fan (*Peking University*)

On gauged linear sigma model

Gauged linear sigma model was a physical model proposed by E. Witten in the early of 90's, which was used by him to explain the mirror symmetry phenomenon. Only up to recent years has this model been seriously considered by mathematicians with the development of Gromov-Witten theory and quantum singularity theory (FJRW theory). In this talk, I will report our progress on this topic.

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Peter Heinzner (*Ruhr University Bochum*)

Kählerian reduction

We will consider a Hamiltonian action of a non compact group G consisting of holomorphic Kähler isometries on a Kählerian manifold X. If the zero fiber $\mathcal{M}$ of the corresponding momentum map is non empty, then the quotient $\mathcal{M}/G$ is know to be a Kähler space. We will show that locally near $\mathcal{M}$ the Kählerian form ω on X has a G-invariant Kähler potential, i.e. it is given in a G-invariant neighborhood by an invariant strictly plurisubharmonic function ρ which determines the momentum map.

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Stanislaw Janeczko (*Warsaw University of Technology*)

Geometric and algebraic restrictions of differential forms

For a smooth manifold M and the space $\Lambda^p(M)$ of all differential p-forms on M the restriction $\omega|_N$ of $\omega \in \Lambda^p(M)$ to a smooth submanifold $N \subset M$ is well defined by the geometry of N. If N is any subset of M then the forms $\alpha + d\beta$, $\alpha \in \Lambda^p(M)$, $\beta \in \Lambda^{p-1}(M)$, where α and β annihilates any p - tuple (and $p-1$ - tuple respectively) of vectors in T_xM, $x \in N$, are called algebraically vanishing on N or having zero algebraic restriction to N. Now the restriction (algebraic restriction) of $\omega \in \Lambda^p(M)$ to N is defined as an equivalence class of ω modulo forms with zero algebraic restriction to N. We study germs of differential forms over singular varieties. The geometric restriction of differential forms to singular varieties is introduced and algebraic restrictions of differential forms with vanishing geometric restrictions, called residual algebraic restrictions, are investigated. Residues of plane curves-germs, hypersurfaces, Lagrangian varieties as well as the geometric and algebraic restriction via a mapping were calculated. This is a joint work with Goo Ishikawa.

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Alexey Karapetyants (*Southern Federal University, Rostov-on-Don*)

On a class of Hausdorff–Berezin operators on the unit disc

We introduce and study the class of Hausdorff-Berezin operators on the unit disc in the Lebesgue p-spaces with Haar measure. We discuss certain algebraic properties of such operators, and also give sufficient, and, in some cases necessary boundedness conditions for such operators. Joint work with Profs. K.Zhu and S.Samko.

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Gulmirza Khudayberganov (*National University of Uzbekistan named after M. Ulugbek, Tashkent*)

Matrix domains in the theory functions of several complex variables

The theory functions of several complex variables or multidimensional complex analysis, despite the various approaches presented in the exposition, currently has a fairly strictly constructed theory. At the same time, many issues of classical (one-dimensional) complex analysis still do not have unambiguous multidimensional analogues. This is due to the complex structure of the multidimensional complex space, the polysemy (overdetermination) of the Cauchy - Riemann conditions, the absence of a universal Cauchy integral formula, etc. In the works of E.Kartan, K.Siegel, Hua Lo-Ken, I.I.Pyatetskiy-Shapiro, B.V.Shabat widely used matrix approach presentation of the theory of multidimensional complex analysis ([1-4]). Here, classical fields and the related problems of the theory of functions and geometry are mainly investigated. The importance of studying classical domains is that they are not reducible, i.e.

these areas are in some sense model areas of multidimensional space. The report presents the latest results in multidimensional complex analysis related to classical domains.

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Sergey Kislyakov (*St. Petersburg Department of Steklov Mathematical Institute of the Russian Academy of Sciences*)

Interpolation for intersections of certain Hardy-type spaces

This is a part of a joint work by the author and K. Zlotnikov

Let (X_0, X_1) be a compatible couple of Banach spaces, and let Y_0, Y_1 be closed subspaces of X_0 and X_1. The couple (Y_0, Y_1) is said to be K-closed in (X_0, X_1) if, whenever $Y_0 + Y_1 \ni x = x_0 + x_1$ with $x_i \in X_i$, $i = 0, 1$, we also have $x = y_0 + y_1$ with $y_i \in Y_i$ and $\|y_i\| \leq C\|x_i\|$, $i = 1, 2$. We recall that K-closedness does occur in the scale of the Hardy spaces on the unit circle (viewed as subspaces of the corresponding Lebesgue spaces), but we are interested in the following two more complicated results (in them we assume that $1 < p < \infty$).

1) The couple $(H^p(\mathbb{T}^2), H^\infty(\mathbb{T}^2))$ is K-closed in the couple $(L^p(\mathbb{T}^2), L^\infty(\mathbb{T}^2))$ (Kislyakov and Xu, 1996).

2) For an inner function θ on the unit circle, the couple $(H^p \cap \theta\overline{H^p}, H^\infty \cap \theta\overline{H^\infty})$ is K-closed in $(L^p(\mathbb{T}), L^\infty(\mathbb{T}))$ (Kislyakov and Zlotnikov, 2018)

Surprisingly, the proofs of these facts are quite similar, signalizing that they may be particular cases of some general statement. Such a statement exists indeed and looks roughly like this. Again, here $1 < p < \infty$.

Theorem. *Let (X, μ) be a space with a finite measure μ, let A and B be w^*-closed subalgebras of $L^\infty(\mu)$, and let C and D be closed subspaces of $L^p(\mu)$ that are moduli over A and B, respectively.* **Under certain additional assumptions***, the couple $(C \cap D, C \cap D \cap L^\infty(\mu))$ is K-closed in $(L^p(\mu), L^\infty(\mu))$.*

The **additional assumptions** say, in particular, that some analogs of the harmonic conjugation operator relative to the algebras A and B have the usual properties, as, for instance, is in the case of w^*-Dirichlet algebras. However, the condition for A and B to be w^*-Dirichlet is too restrictive (in particular, we do not insist that a multiple of μ represent some multiplicative linear functional on either A or B). Also, note that the proofs of statements 1) and 2) known previously relied upon the fact that, in those settings, the corresponding harmonic conjugations (or Riesz projections) were classical singular integral operators. In particular, the two proofs started with employing Calderón–Zygmund decomposition, which is not available in the generality adopted in the theorem. Also, some assumptions on the mutual position of the annihilators of C and D are required (in the context of statements 1 and 2, these assumptions are satisfied trivially).

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Ryoichi Kobayashi (*Nagoya University*)

Rigidity of compact holomorphic curves in compact complex parallelizable manifolds $\Gamma\backslash\mathrm{SL}(2,\mathbb{C})$ ***and its geometric applications***

Let $\Gamma \subset \mathrm{SL}(2,\mathbb{C})$ be a cocompact lattice and $X = \Gamma\backslash\mathrm{SL}(2,\mathbb{C})$ the associated compact complex parallelizable manifold. We show that any non-constant holomorphic map $f : M \to X$ from a compact Riemann surface M into X decomposes as $f = t \circ h \circ \alpha$, where $\alpha : M \to \mathrm{Alb}(M)$ is the Albanese map, $h : \mathrm{Alb}(M) \to X = \Gamma\backslash\mathrm{SL}(2,\mathbb{C})$ has its image in a maximal torus $T = \Gamma \cap A\backslash A \cong \mathbb{Z}\backslash\mathbb{C}^*$ in X (A being a maximal torus in $\mathrm{SL}(2,\mathbb{C})$) defining an algebraic group homomorphism $h : \mathrm{Alb}(M) \to T = (A \cap \Gamma)\backslash A$, and finally t is a right translation by some element of $\mathrm{SL}(2,\mathbb{C})$. The proof is based on Bishop's measure theoretic criterion of analyticity of sets combined with a simple observation in hyperbolic geometry.

I will discuss some applications of this rigidity.

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Vladislav Kravchenko (*Southern Federal University, Rostov-on-Don*)

Construction of transmutation operators and application to direct and inverse spectral problem

A new approach for solving the classical direct and inverse Sturm-Liouville problems on finite and infinite intervals is presented. It is based on the Gel'fand-Levitan-Marchenko integral equations and recent results on the functional series representations for the transmutation (transformation) operator kernels [1-5]. New representations of solutions to Sturm-Liouville equation are obtained enjoying the following feature important for practical applications. Partial sums of the series admit estimates independent of the real part of the square root of the spectral parameter which makes it especially convenient for the approximate solution of spectral problems. Numerical methods based on the proposed approach for solving direct problems allow one to compute large sets of eigendata with a nondeteriorating accuracy. Solution of the inverse problems reduces directly to a system of linear algebraic equations. In the talk some numerical illustrations will be presented.

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Ngaiming Mok (*The University of Hong Kong*)

Uniformization Problems on Subvarieties of Finite-Volume Quotient Spaces of Bounded Symmetric Domains

By the Uniformization Theorem a compact Riemann surface of genus ≥ 2 is uniformized by the unit disk Δ and equivalently by the upper half plane $\mathcal{H}$. $\mathcal{H}$ is also the universal covering space of the moduli space of elliptic curves equipped with an appropriate level structure. In Several Complex Variables, the Siegel upper half plane $\mathcal{H}_g, g \geq 1$ is an analogue of $\mathcal{H} = \mathcal{H}_1$, and it is the universal covering space of moduli spaces of polarized Abelian varieties with appropriate level structures. $\mathcal{H}_g$ belongs, up to biholomorphic equivalence, to the set of bounded symmetric domains, on which a great deal of mathematical research is taking place. Especially, finite-volume quotients of bounded symmetric domains Ω, which are naturally quasi-projective varieties, are objects of immense interest to Several Complex Variables, Algebraic Geometry, Arithmetic Geometry and Number Theory, and an important topic is the study of covering spaces of algebraic subsets of such quasi-projective varieties. While a lot has already been achieved in the case of Shimura varieties by means of methods of Diophantine Geometry, Model Theory, Hodge Theory and Complex Differential Geometry, techniques for the general case of not necessarily arithmetic quotients Ω/Γ have just begun to be developed. We will explain a differential-geometric approach leading to various characterization results for totally geodesic subvarieties of finite-volume quotients Ω/Γ without the assumption of arithmeticity. Especially, we will explain how the study of holomorphic isometric embeddings of the Poincaré disk and more generally complex unit balls into bounded symmetric domains can be further developed to derive uniformization theorems for bi-algebraic varieties and more generally for the Zariski closure of images of algebraic sets under the universal covering map.

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Alfonso Montes-Rodríguez (*University of Seville*)

Uniform sampling along characteristics for solutions of the telegraph system

For each function $a : \mathbb{R} \mapsto \mathbb{C}$ with integrable modulus on $\mathbb{R}$, we define the *exponential telegraphic* function as

$$a_{\mathbb{T}}(x,y) := \int_{\mathbb{R}} a(t)\ \exp\left(ixt + iy/t\right)\, dt\ , \quad x, y \in \mathbb{R}\ .$$

Every exponential telegraphic function is a continuous solution on $\mathbb{R}^2$ of the partial differential equation $U_{xy} + U = 0$ with two independent real variables x, y. Conversely, for each continuous solution w of the equation $U_{xy} + U = 0$ on a convex compact subset K in $\mathbb{R}^2$ with nonempty interior, there exists an exponential telegraphic function $a_{\mathbb{T}} = a_{\mathbb{T}}(w, K)$ which coincides with w on K whenever w_x and w_y are continuous on K. Exponential telegraphic functions have first been studied in 2011, see [1] where it is proved that each such function can be recoverable sampled at the points $(0, \pi n), (\pi n, 0),\ n \in \mathbb{Z} := \{..., -1, 0, 1, ...\}$, lying on two characteristics $x = 0$ and $y = 0$ of the equation $U_{xy} + U = 0$. In other words, it follows from $a_{\mathbb{T}}(\pi n, 0) = a_{\mathbb{T}}(0, \pi n) = 0$, $n \in \mathbb{Z}$, that $a_{\mathbb{T}}(x, y) = 0$ for every $x, y \in \mathbb{R}$. In this work, we provide a new proof of the fact that $a_{\mathbb{T}}(\pi n, 0) = a_{\mathbb{T}}(0, -\pi n) = 0$ for all $n \in \mathbb{N}_0 := \{0, 1, 2, ...\}$, implies $a_{\mathbb{T}}(x, -y) = 0$ for each $x, y \geq 0$ (cp. [2]), which means possibility to restore each exponential telegraphic function in the quadrant $[0, +\infty) \times (-\infty, 0]$ by its values at the points $(0, -\pi n), (\pi n, 0),\ n \in \mathbb{N}_0$. We apply these results to continuously differentiable one time by each variable solutions $v(t, x)$ and $i(t, x)$ of the telegraph system

$$\begin{cases} i_x(t,x) + C \cdot v_t(t,x) + G \cdot v(t,x) = 0\,, & R - \text{resistance}, \quad L - \text{inductance}, \quad D := LG - CR \neq 0, \\ v_x(t,x) + L \cdot i_t(t,x) + R \cdot i(t,x) = 0\,, & C - \text{capacitance}, \quad G - \text{leakance}, \quad t \geq 0,\ x \in \mathbb{R}, \end{cases}$$

with the additional restriction of the existence $T > 0$ satisfying $v(t,0) = i(t,0) = 0$, $t \geq T$. It follows that such v and i in the angle $|x| \leq t/\sqrt{LC}$, $t \geq 0$, $x \in \mathbb{R}$ between the two characteristics $x = \pm t/\sqrt{LC}$ are uniquely determined by the values of v or i at the points $(2\pi nLC/|D|\,,\ \pm 2\pi n\sqrt{LC}/|D|)$, $n \in \mathbb{N}_0$, lying on these characteristics.

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Oleg Mushkarov (*Institute of Mathematics and Informatics of the Bulgarian Academy of Sciences, Sofia*)

Partial integrability of almost complex structures

In this talk we will review some old and new results on the problem for local and global existence of holomorphic functions on almost complex manifolds. The following topics will be considered: Existence of holomorphic functions and IJ-bundles, Hypersurfaces in $\mathbb{R}^7$, Nearly Kähler manifolds, Homogeneous almost complex spaces, Partial integrability of the Atiyah-Hitchin-Singer and Eells-Salamon almost complex structures on twistor spaces. Some open problems will also be discussed.

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Il'dar Musin (*Institute of Mathematics with Computer Centre of Ufa Federal Research Centre of Russian Academy of Sciences*)

Linear operators on Fock type spaces

A weighted Hilbert space F^2_φ of entire functions of n variables will be considered in the talk. It is constructed with a help a weight function φ on $\mathbb{R}^n$. φ is a semicontinuous from below function on $\mathbb{R}^n$ depending on modules of variables, growing at infinity faster than $a\ln(1+\|x\|)$ for each positive a and such that its restriction on $[0,\infty)^n$ is nondecreasing in each variable. Properties of the space F^2_φ will be described. The main part of the talk is devoted to concrete operators acting on F^2_φ (among them Toeplitz operators, weighted composition operators). Under some additional conditions on φ (convexity, growth conditions) the space of the Laplace transforms of linear continuous functionals on F^2_φ is described.

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Semen Nasyrov (*Kazan Federal University*)

Nuttall's decomposition of a three-sheeted Riemann surface of genus one

We investigate the structure of a decomposition of the Riemann surface $\mathfrak{R}$ of the function $\sqrt[3]{(z-a)(z-b)(z-c)}$ into 3 sheets. The decomposition is specified by an Abelian integral with logarithmic singularities over the infinite points of $\mathfrak{R}$. In the case, when the triangle with vertices a, b, and c is close to a regular one, the problem was studied by A. I. Aptekarev and D. N. Tulyakov. We consider the general case. The main attention is paid to investigation of the problem, if the critical points of the Abelian differential lie on the borders of the sheets.

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Tuen-Wai Ng (*The University of Hong Kong*)

The squeezing functions on planar domains

In 2012, Deng, Guan and Zhang introduced the concept of squeezing function for a bounded domain. This function of several complex variables is a very interesting new biholomorphic invariant. In this talk, we shall focus on the squeezing functions of some simple planar domains and explain how some classical tools in univalent function theory can be used to study these functions.

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Anatol Odzijewicz (*University of Bialystok*)

Quantization and groupoid of partially invertible elements of W^*-algebra

We present a method of quantization of one-parameter groups of automorphisms of a G-principal bundle $\pi : P \to M$ with a fixed connection form $\alpha \in \Gamma^\infty(P)$. This method is based on the morphism of a gauge groupoid $\frac{P\times P}{G} \rightrightarrows M$ into the groupoid $\mathcal{G}(\mathfrak{M}) \rightrightarrows \mathcal{L}(\mathfrak{M})$ of partially invertible elements of a W^*-algebra $\mathfrak{M}$. We also show that one can consider the Kirillov-Kostant-Souriau geometric quantization as well as coherent state quantization as some particular cases of the proposed approach.

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Martin Schlichenmaier (*University of Luxembourg*)

N ***point Virsasoro algebras are multi-point Krichever-Novikov type algebras.***

We show how the recently again discussed N-point Witt, Virasoro, and affine Lie algebras are genus zero examples of the multi-point versions of Krichever–Novikov type. These multi-point versions were introduced and studied by Schlichenmaier. Using this more general point of view, useful structural insights and an easier access to calculations can be obtained. The concept of almost-grading will yield information about triangular decompositions which are of importance in the theory of representations. The algebra of functions, vector fields, differential operators, current algebras, affine Lie algebras, Lie superalgebras and their central extensions show up. As special example the three-point case is given.

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Valery Serov (*University of Oulu*)

Scattering and Spectral Problems for Perturbations of Multidimensional Biharmonic Operator

The subject of this work concerns to the classical inverse scattering and inverse spectral problems. The inverse scattering problem can be formulated as follows: do the knowledge of the far field pattern uniquely determines (and how) the unknown coefficients of given differential operator? Saito's formula and uniqueness result, as well as the reconstruction of singularities, are obtained for the scattering problems (see [1], [2]). The inverse spectral problem can be formulated as follows: do the Dirichlet eigenvalues and the derivatives (of which order?) of the normalized eigenfunctions at the boundary determine uniquely the coefficients of the corresponding differential operator? In the present work we show that the knowledge of the discrete Dirichlet spectrum and some special derivatives up to the third order of the normalized eigenfunctions at the boundary uniquely determine the coefficients of the operator of order 4 which is the second order perturbation of the biharmonic operator (see [3]). Usually in the literature is assumed the knowledge of the Dirichlet-to-Neumann map which uniquely determines the unknown coefficients. In the comparison with this we prove (in addition) that the Dirichlet-to-Neumann map can be uniquely determined by the spectral data.

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Alexandre Sukhov (*University of Lille; Institut of Mathematics with Computing Centre - Subdivision of the Ufa Research Centre of RAS*)

Some aspects of analysis on almost complex manifolds

Analysis on almost complex manifolds became a rapidly growing area of Several Complex Variables after important works of M.Gromov and S.Donaldson. In this talk we discuss several aspects of this theory: boundary value problems for pseudoholomorphic curves, the pluripotential theory and its applications, the boundary behavior of $\overline{\partial}$-subsolutions.

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August Tsikh (*Siberian Federal University, Krasnoyarsk*)

Convexity properties for complements of analytic sets and their amoebas

Images on the Reinhardt diagram for subsets of the complex space plays important role in the complex analysis. For analytic subset images, it is convenient to consider the diagram in the logarithmic scale; in this case, the image is called the amoeba of the analytic set. In the talk, we prove that the well-known k-pseudoconvexity property for the complement of the analytic set corresponds to the Gromov-Lefshets k-convexity for the amoeba complement.

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Siye Wu (*National Tsing Hua University, Taiwan*)

Deformation of the prequantum action on a symplectic manifold

Using Fedosov's construction in deformation quantisation, we propose a family of star products associated to complex polarisations on a symplectic manifold and we construct the action of a function on a section of the prequantum line bundle.

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Chungchun Yang (*Nanjing University*)

Nevanlinna's value distribution theory, its applications and conjectures

Some new and old problems or conjectures arisen from the studies of Nevanlinna's value distribution theory and its applications to factorization, complex dynamics, and functional equations of meromorphic functions are surveyed, for further investigations. Most of the conjectures were posed by the speaker and his co-workers two or three decades ago, and the latest one is the following conjecture:

Let f be a transcendental entire function and k be a given integer. If $ff^{(k)}$ is periodic, then f itself must be periodic.

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Akira Yoshioka (*Tokyo University of Science*)

Non-formal star product and star functions

Star product is considered as a generalization of the Moyal product, which is a one parameter deformation of ususal multiplication of functions and is associative. The star products are also parametrized by matrices giving non-commutative or commutative algebraic structures on function space according to the matrices.

In this talk, we will deal with non-formal star products on complex space $\mathbb{C}^n$. For polynomials on $\mathbb{C}^n$, replacing usual multiplication by that of star product gives star polynomials, and similarly for power series of entire functions on $\mathbb{C}^n$ we obtain star functions. The star funtions of entire function have singularities in general.

We give a review on the star products and star functions, and discuss some problems. We consider mainly concrete examples of star functions.

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Xiangyu Zhou (*Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing*)

Some recent results on transcendental techniques in complex geometry

We'll present some our recent solutions of an optimal L^2 extension problem and Demailly's strong openness conjecture on multiplier ideal sheaves, including the backgrounds, applications and further developments.

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